Cylindrical Algebraic Decomposition for Nonlinear Arithmetic Problems

Gereon Kremer

PhD defense talk

12.03.2020

Grateful for the cooperation with and advise from:
Erika Ábrahám, Jens Brandt, Christopher Brown, Florian Corzilius, James H. Davenport, Matthew England, Vijay Ganesh, Rebecca Haehn, Marcel Hark, Einar Broch Johnsen, Sebastian Junges, Viktor Levandovskyy, Jacopo Mauro, Jasper Nalbach, Stefan Schupp, Tarik Viehmann, and many more.

Formal verification (with satisfiability modulo theories)

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Abstract
&
Formalize

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CBMC APROVE CPA✓ Vcc Dafny

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Dafny

Encode to SMT

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Encode to SMT

MathSAT 5
veriT
Yices2
CVC4
Z3
SMT RAT

Formal verification (with satisfiability modulo theories)

Abstract & Formalize

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Encode to SMT

Interpret results

MathSAT 5

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Formal verification (with satisfiability modulo theories)

Abstract & Formalize

Encode to SMT

Interpret results

SMT formulae:

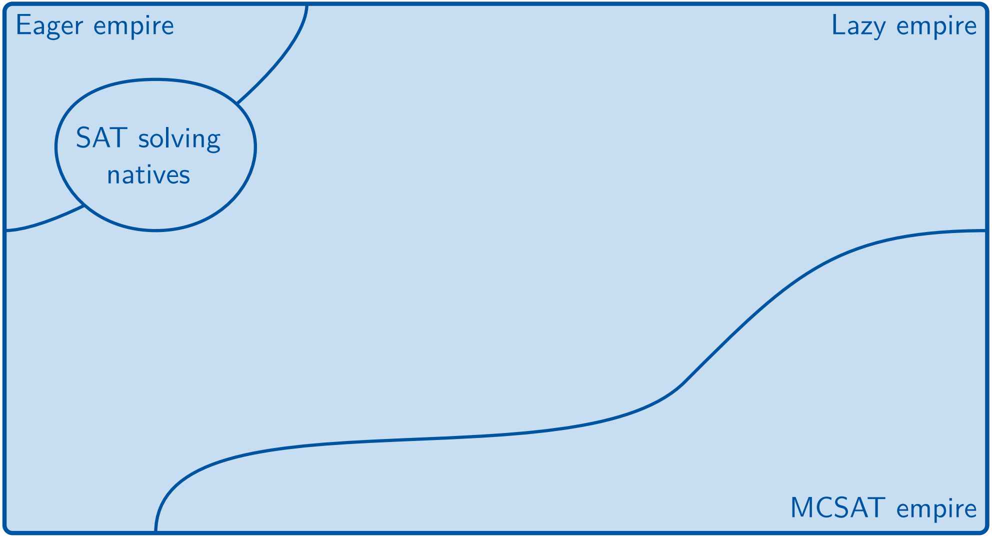
$$\exists x_1, \ldots, x_n \in \mathbb{R}.\ \varphi(x_1, \ldots, x_n)$$

Theory constraints:

$$p \sim 0 \qquad p \in \mathbb{Z}[x_1, \ldots, x_n]$$

In SMT-LIB as `QF_NRA` since 2013

World Map of SMT



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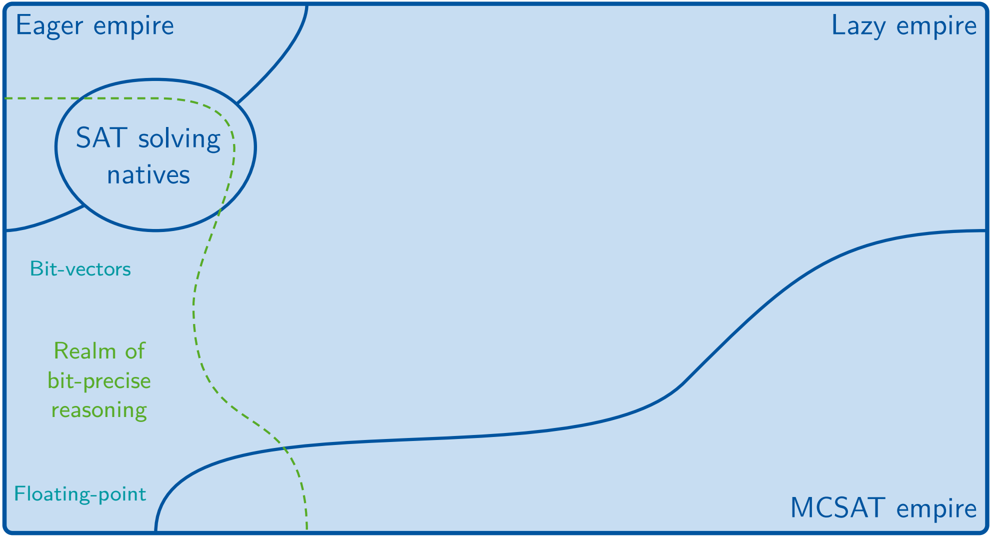

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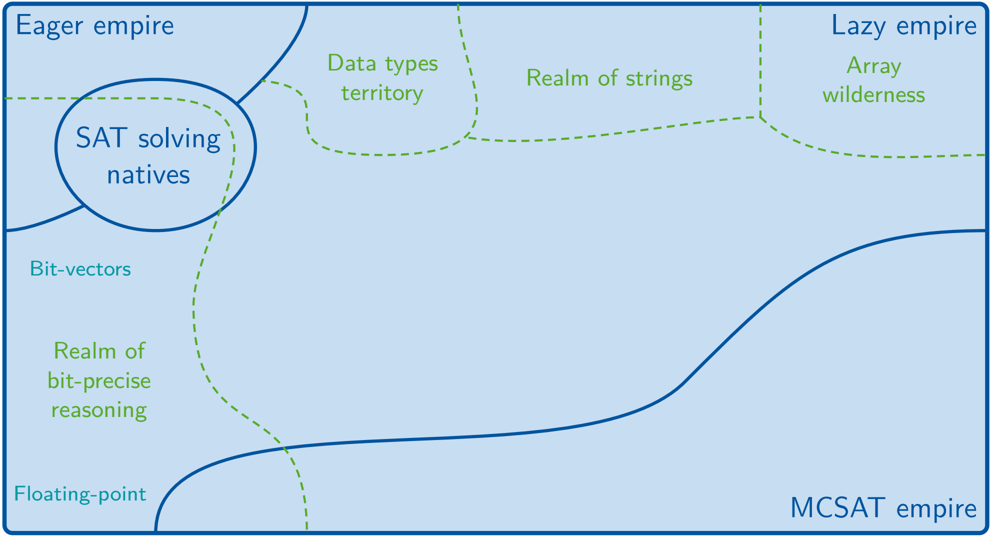

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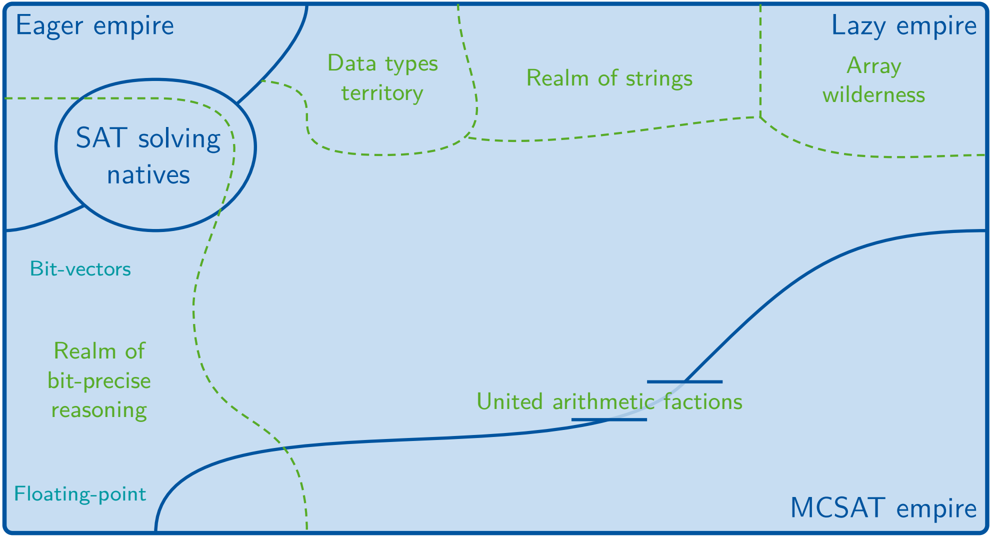

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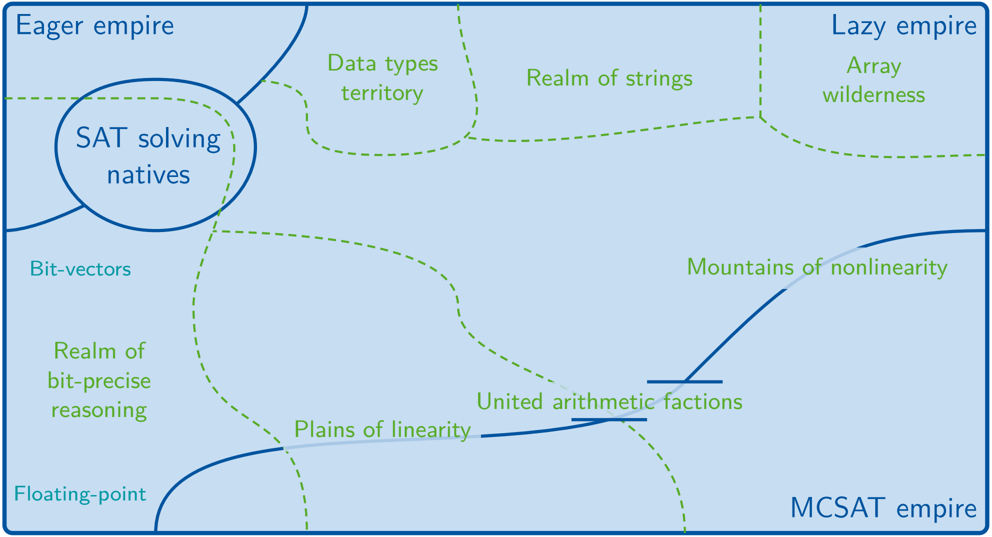

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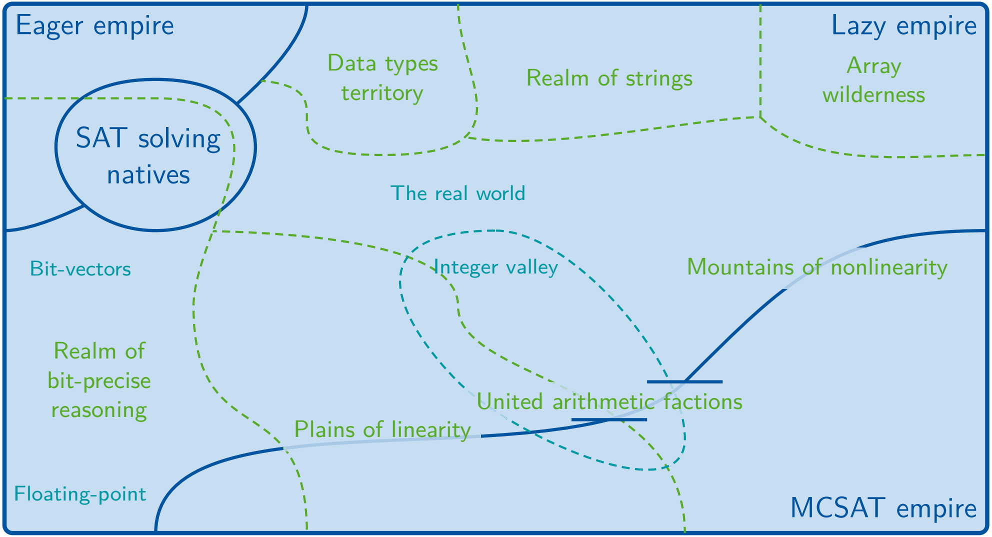

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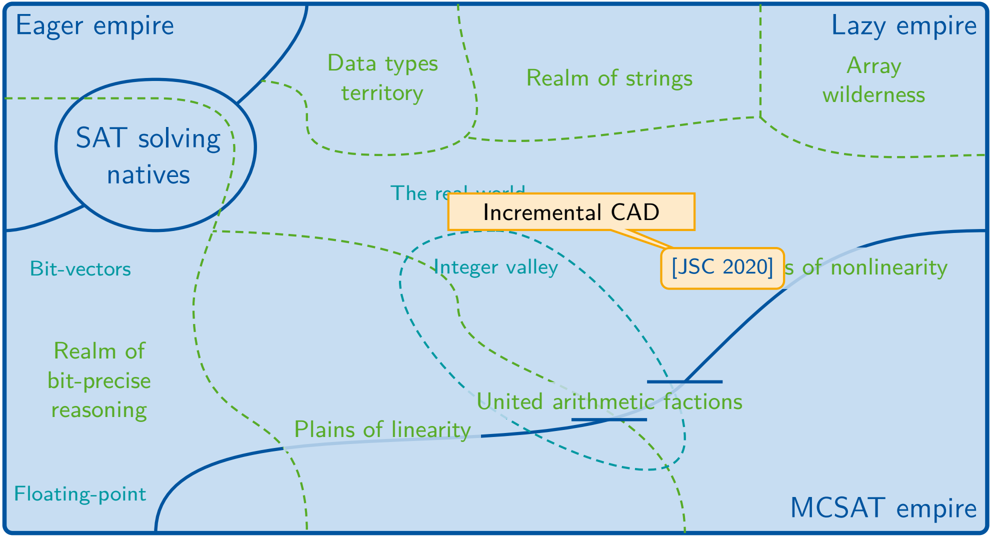

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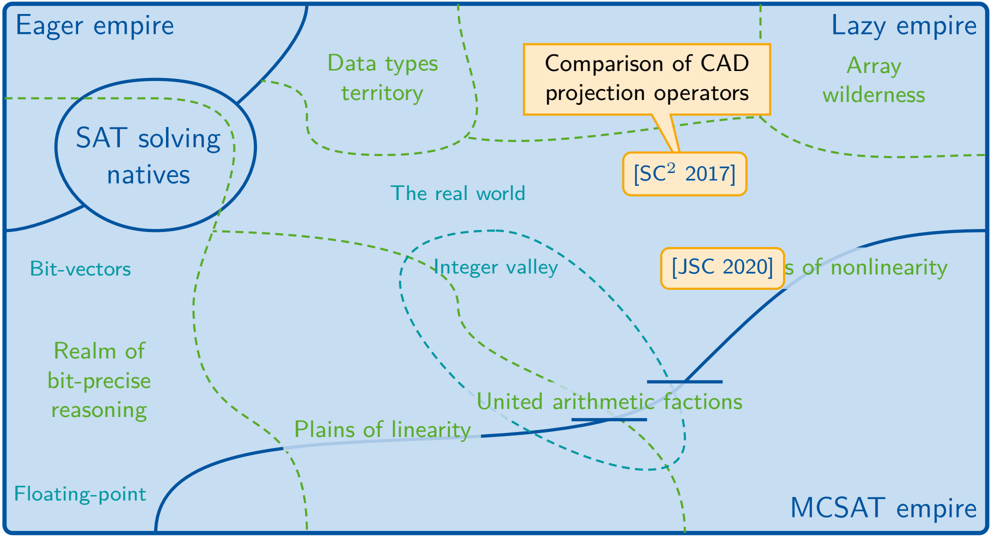

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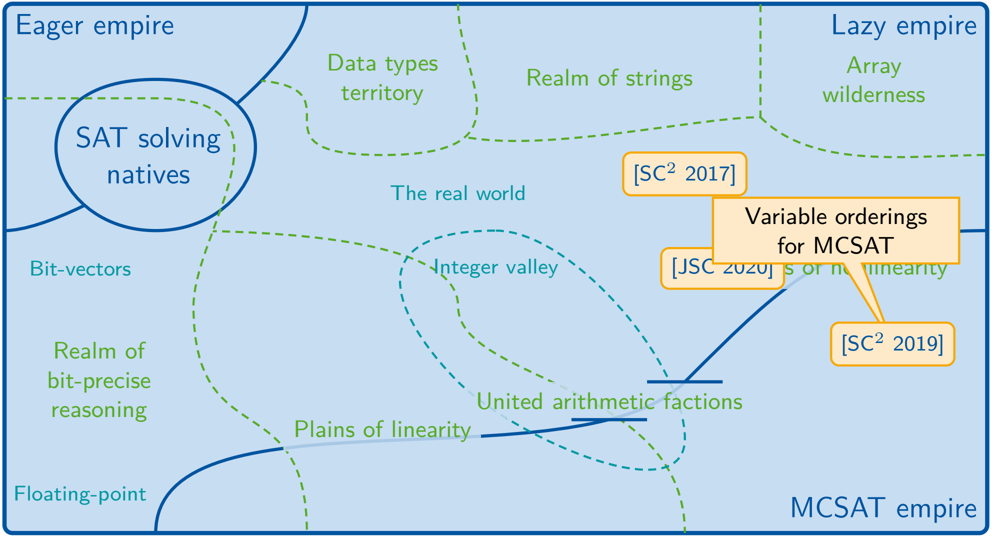

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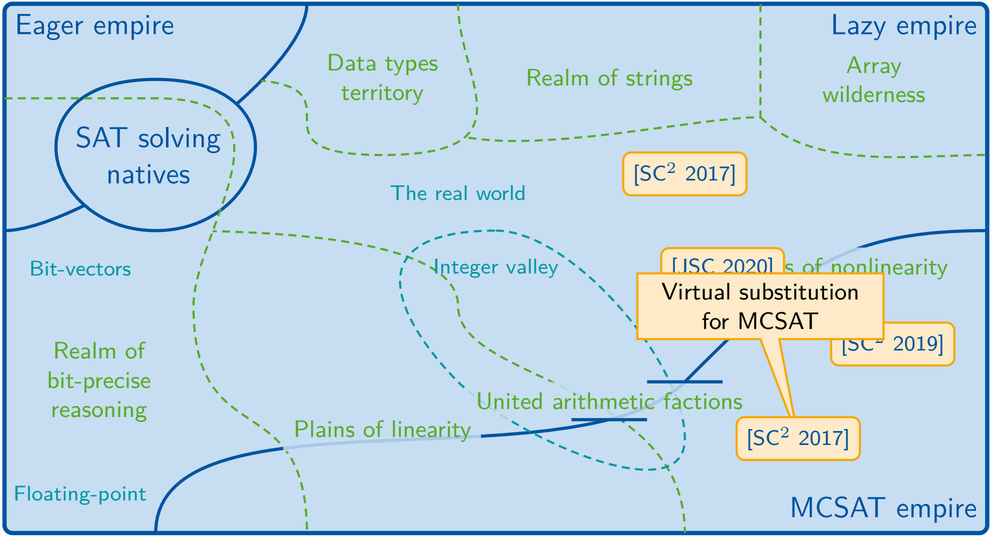

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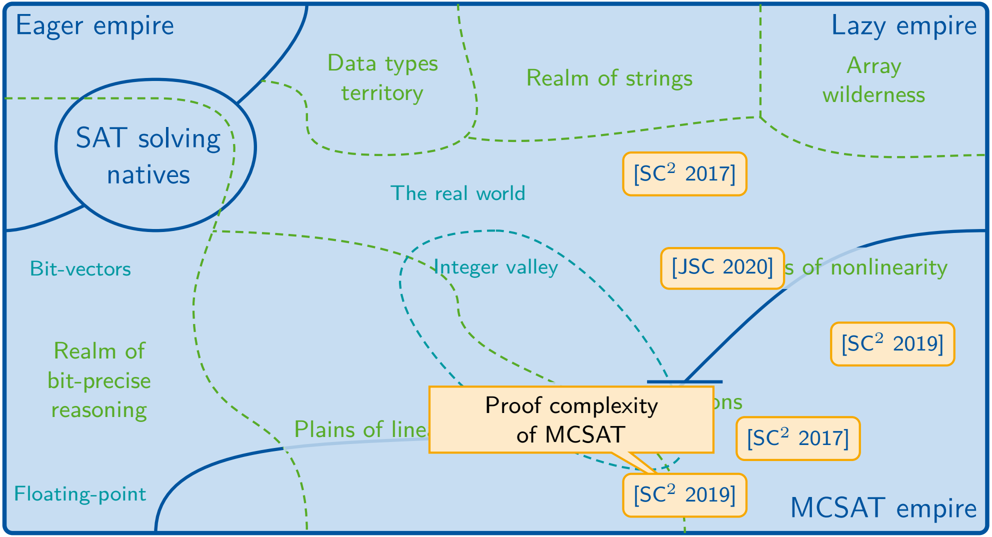

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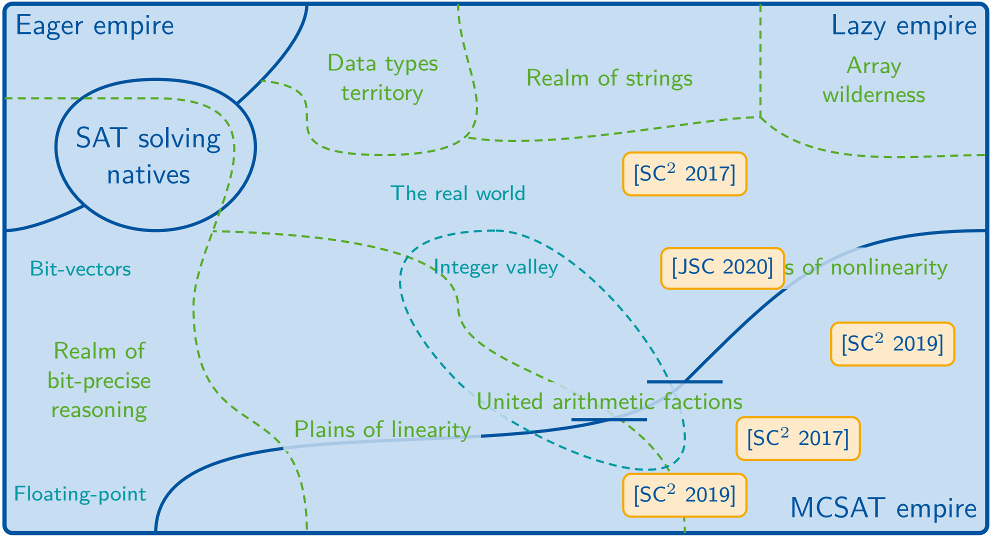
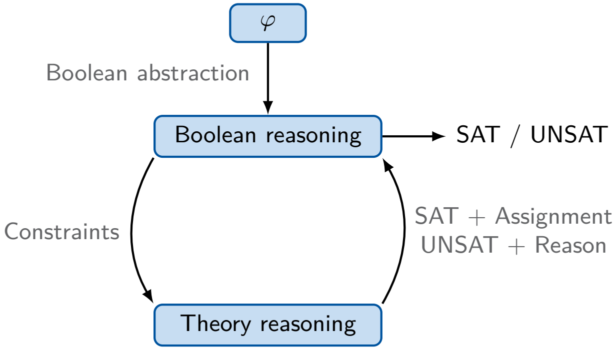


Lazy SMT solving



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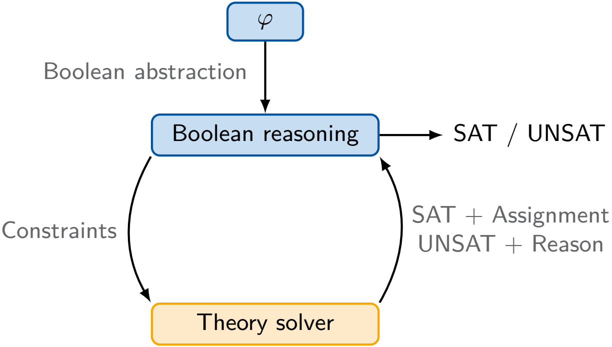
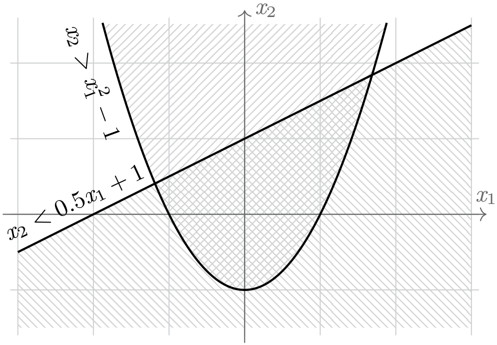


Core ideas of CAD

- Input: set of constraints



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- Input: set of constraints
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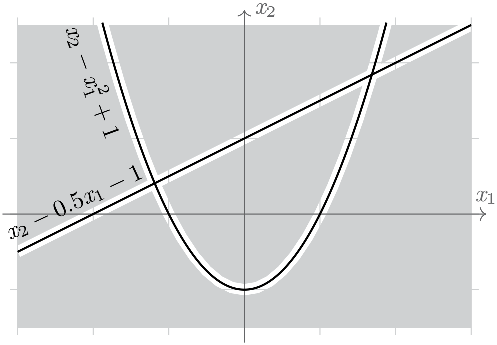
- Identify sign-invariant regions



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- Sign-invariance (of polynomials) implies truth-invariance (of the formula)
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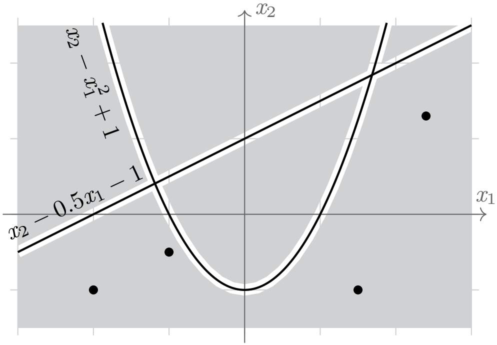

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- Input: set of constraints
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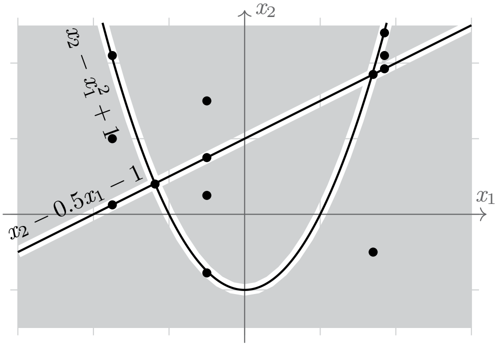
- All samples in one region are equivalent
- Construct one sample per region
- Evaluate samples on constraints



CAD in a nutshell

Set of constraints

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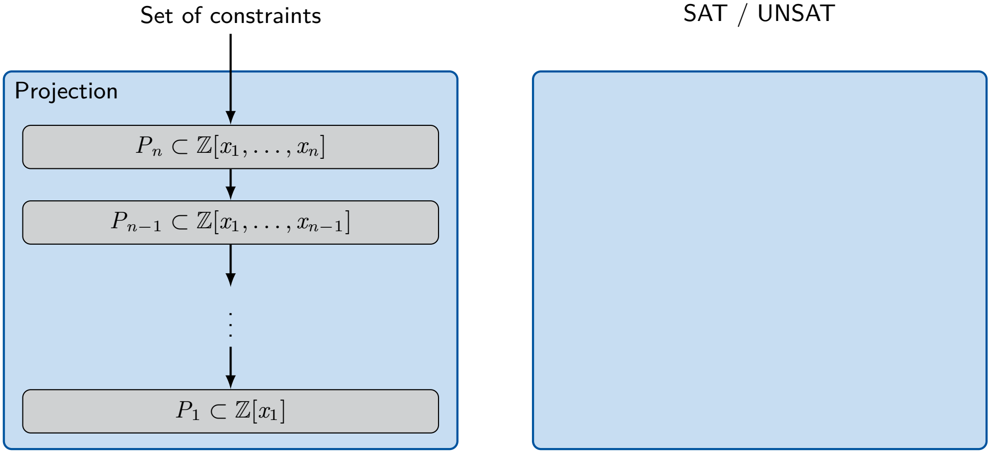
[Collins 1975]

CAD in a nutshell



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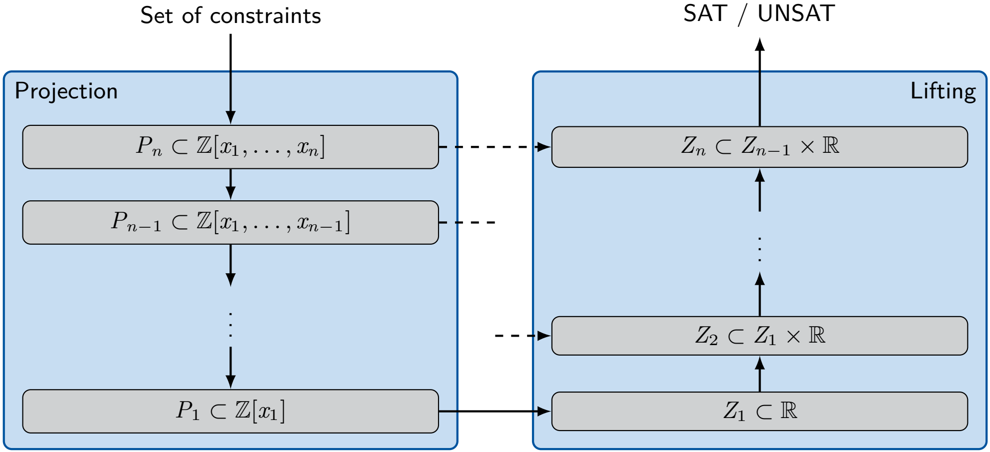

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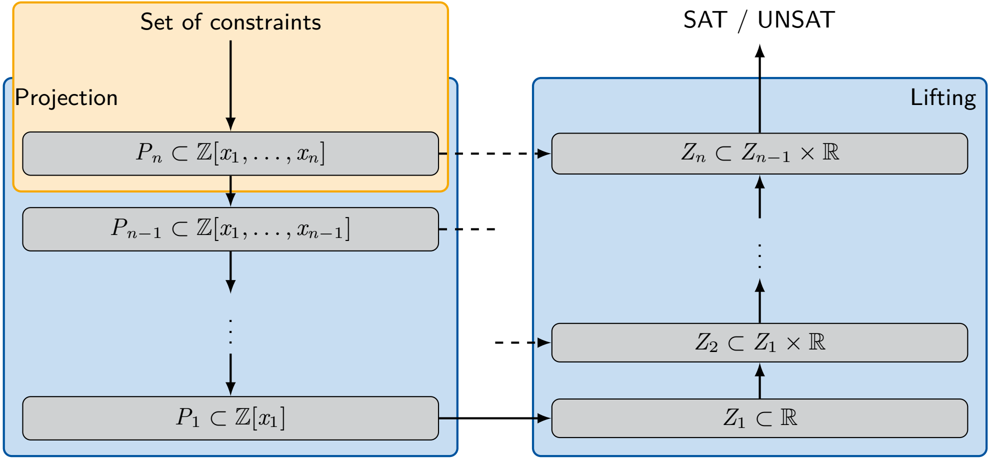

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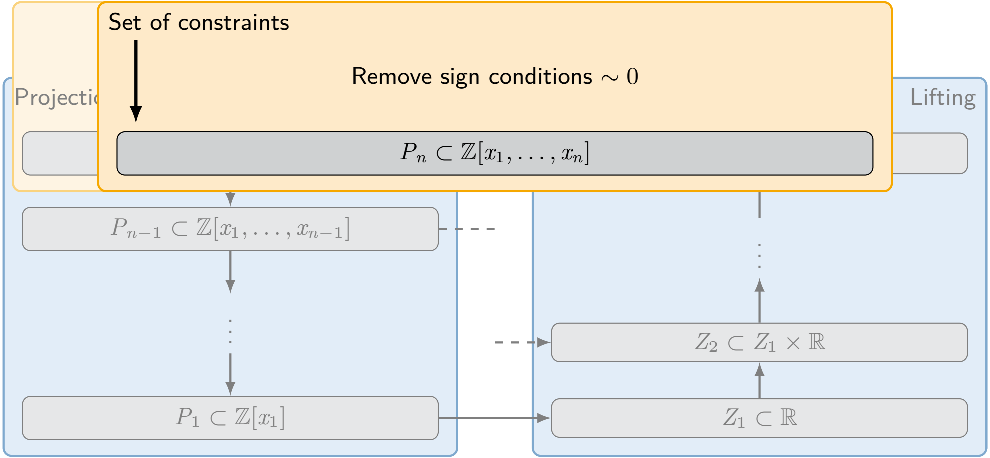

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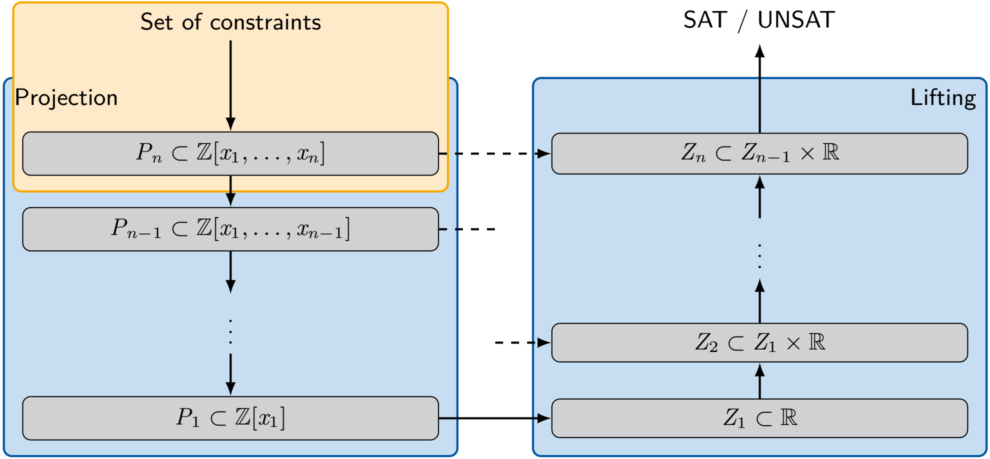

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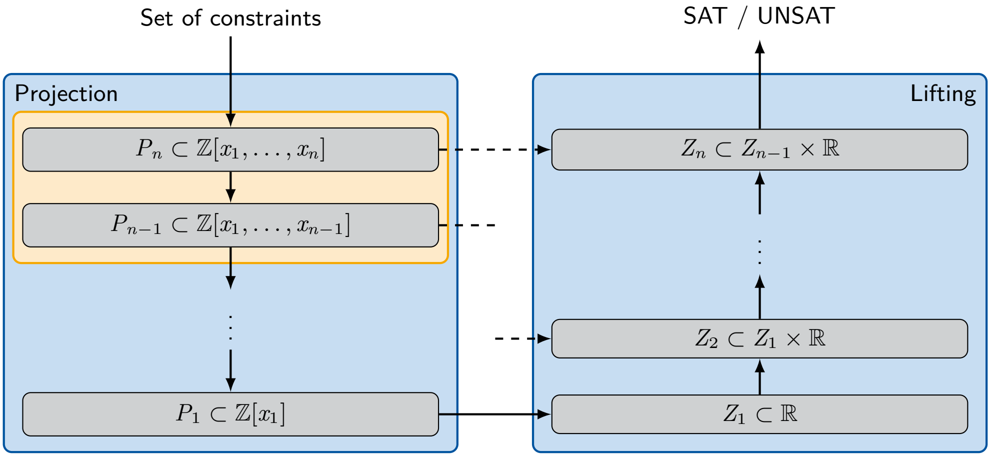

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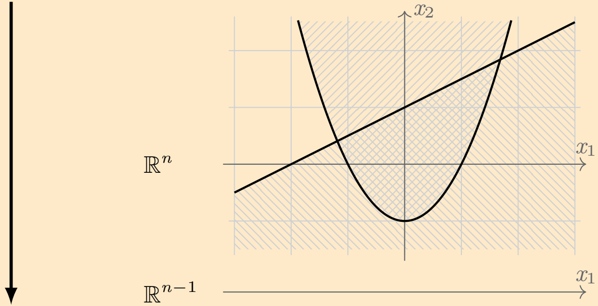
CAD in a nutshell

Projecti

Lifting

Projection

$$P_n \subset \mathbb{Z}[x_1, \ldots, x_n]$$



$$P_{n-1} \subset \mathbb{Z}[x_1, \ldots, x_{n-1}]$$

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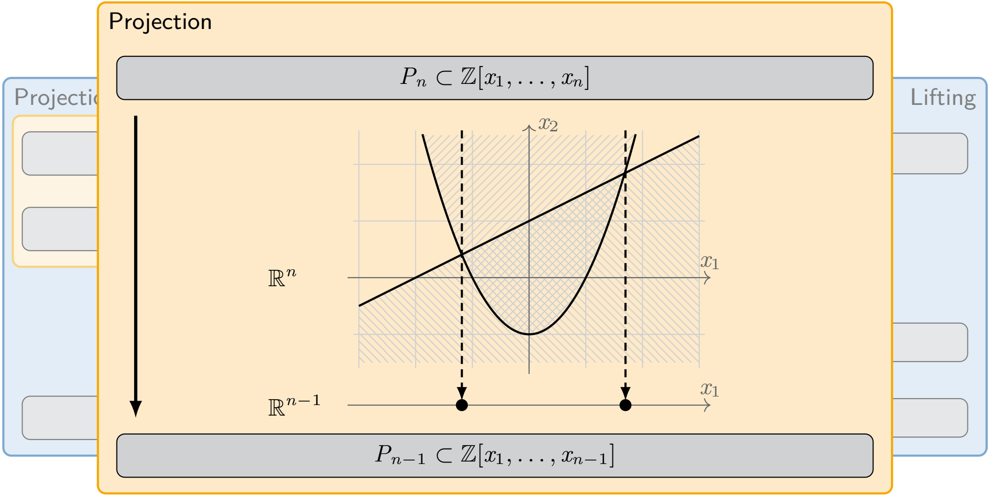

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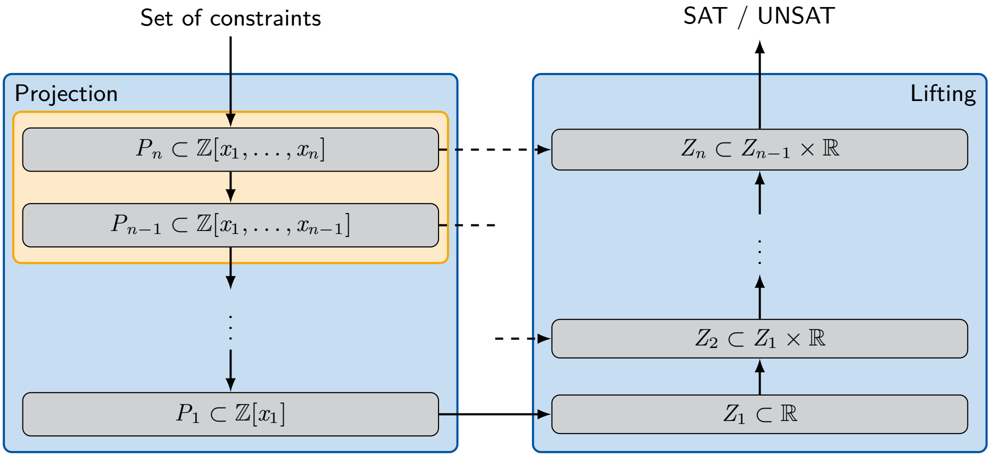

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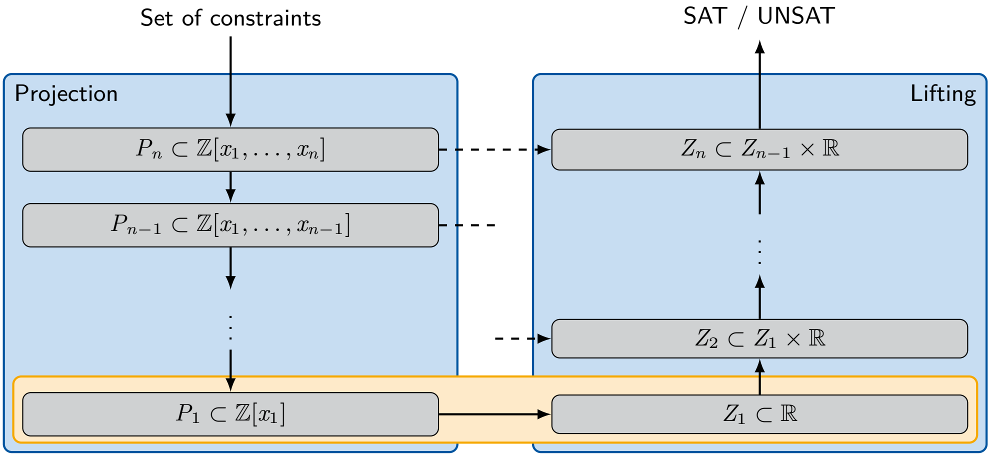

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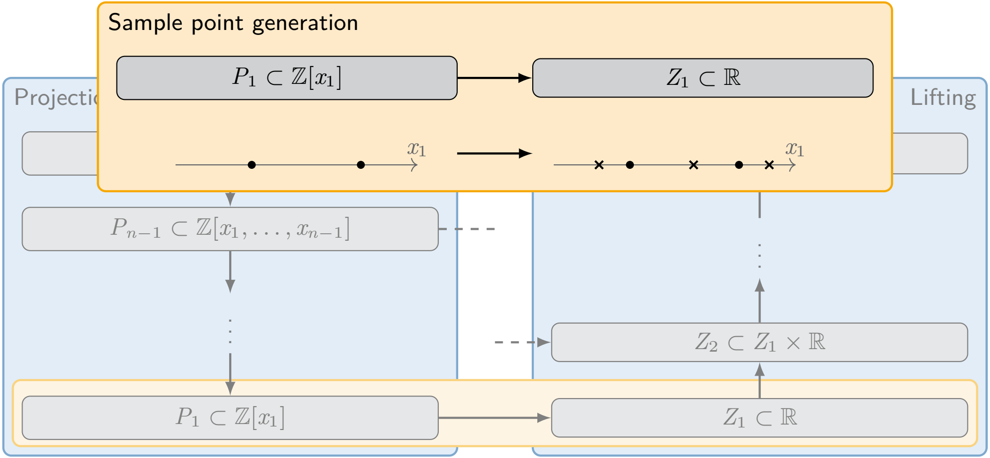

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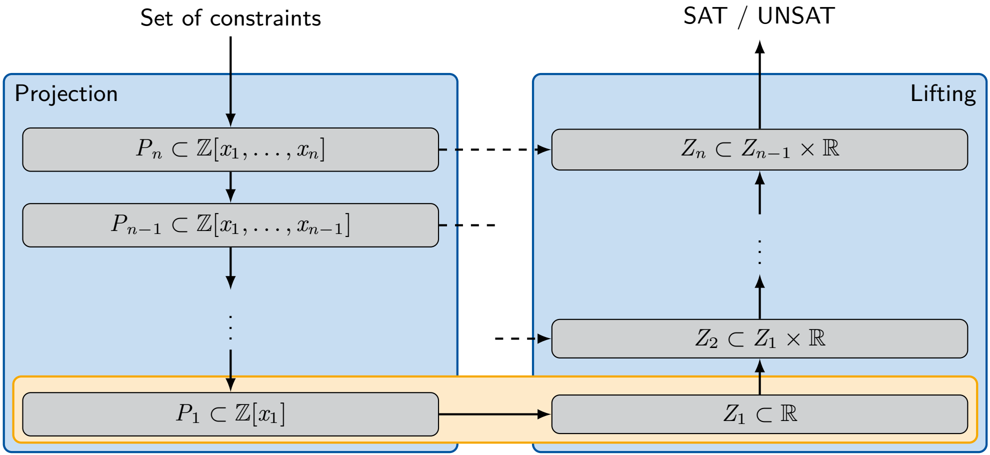

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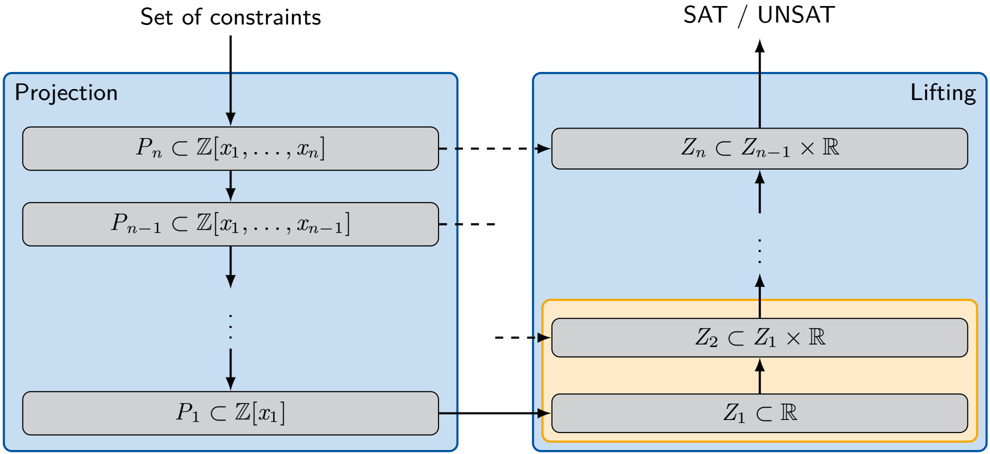

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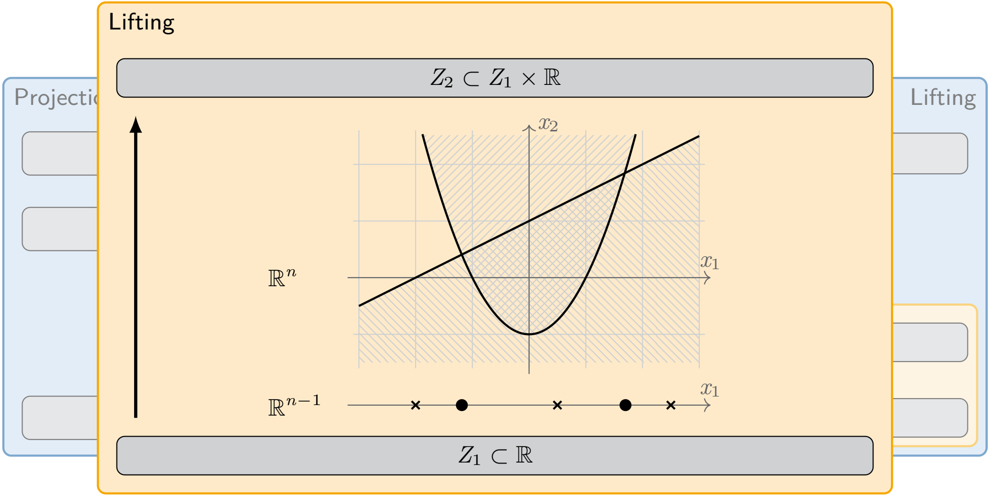

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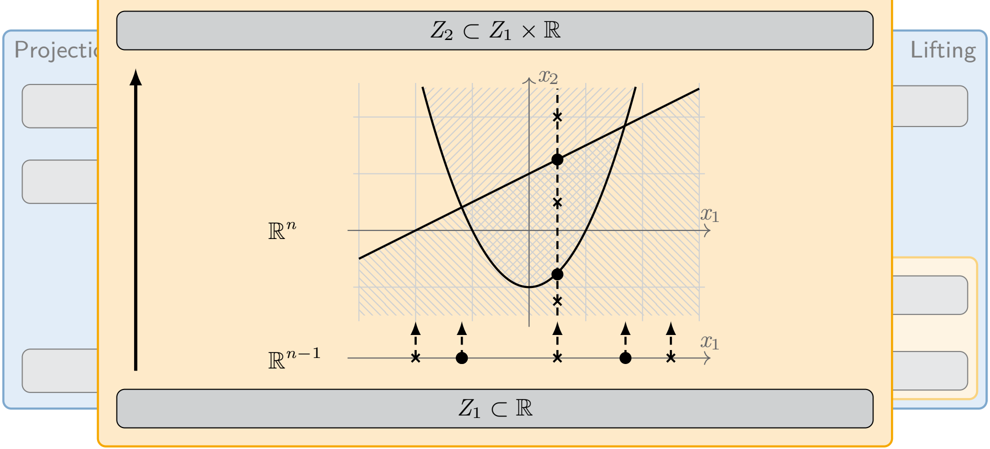

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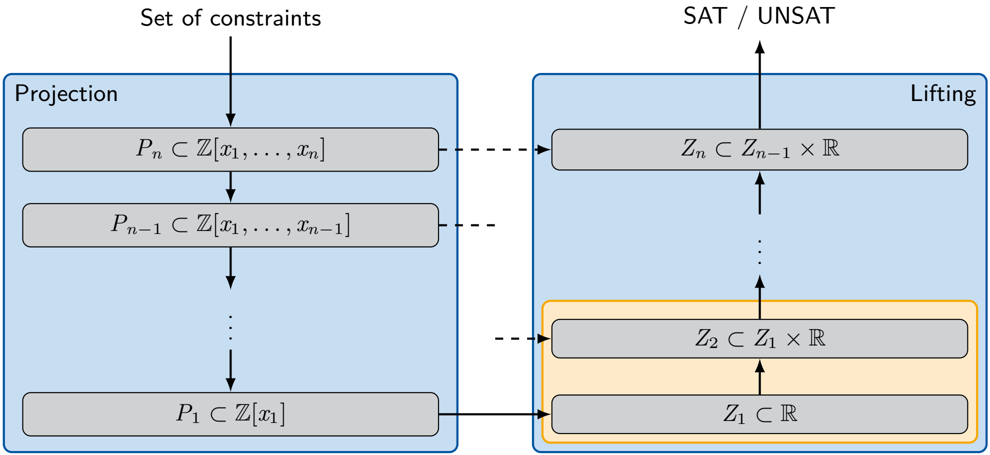

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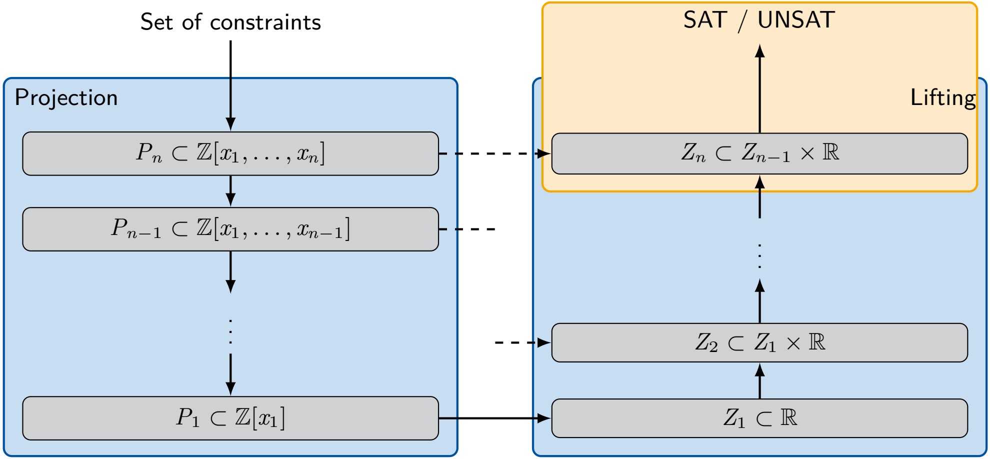

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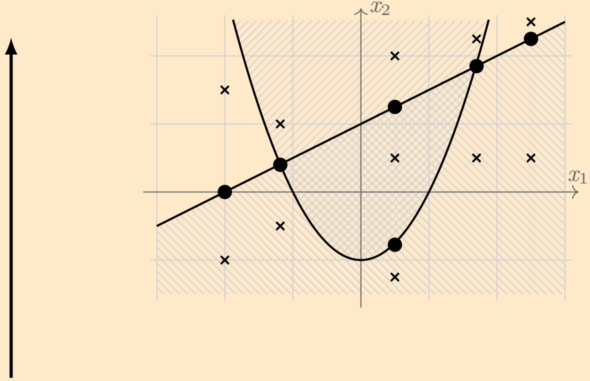

$Z_n \subset Z_{n-1} \times \mathbb{R}$

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CAD in a nutshell

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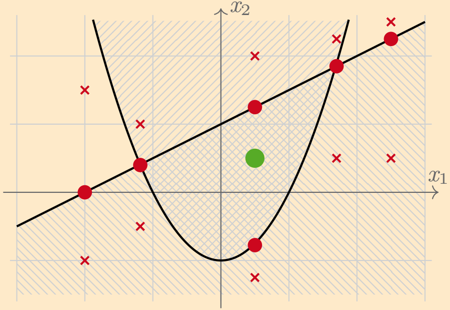
SAT



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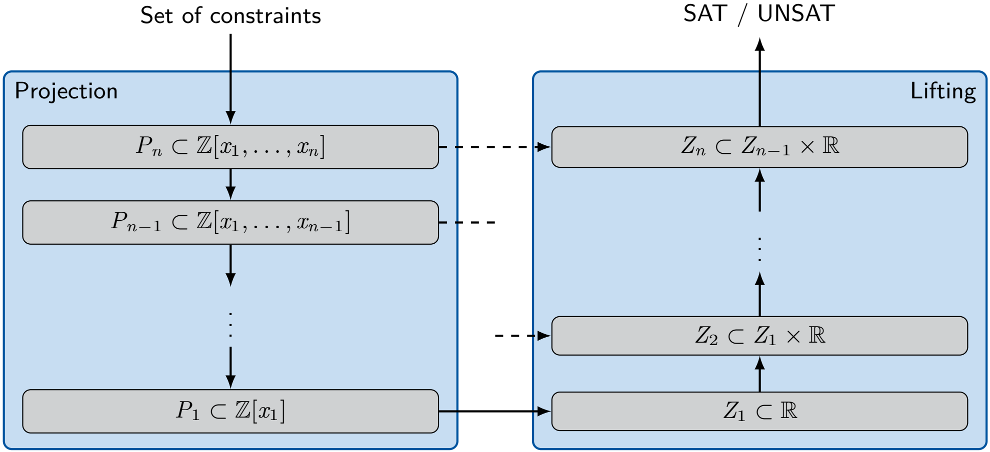

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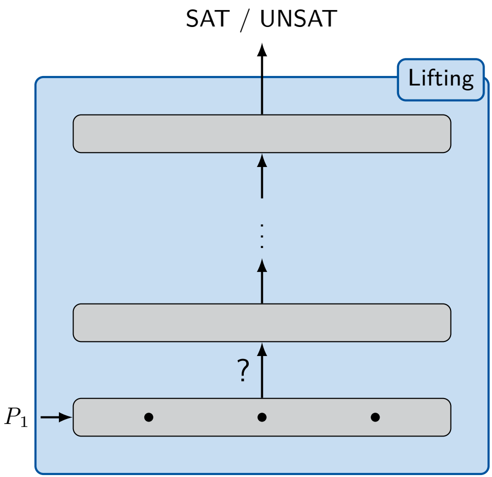
Our goal: make CAD as efficient as possible for SMT

- Incrementality: reuse previously computed results
- Backtracking: remove part of the input efficiently
- Reasons for unsatisfiability: small infeasible subsets

Incremental lifting

[JSC 2020]



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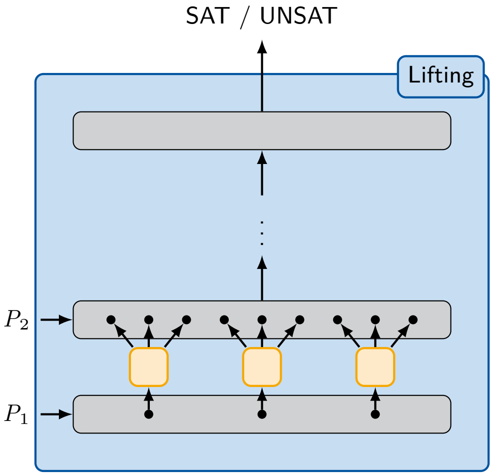

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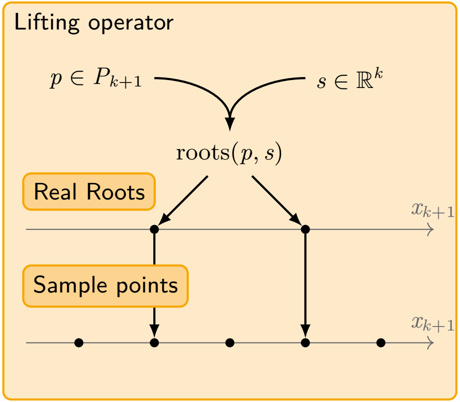

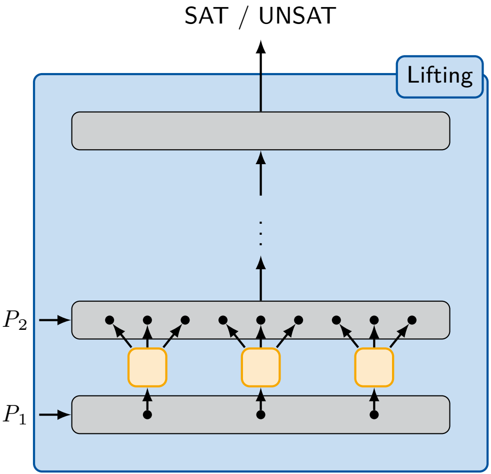

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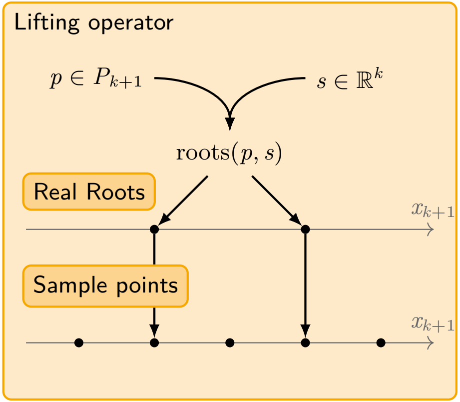

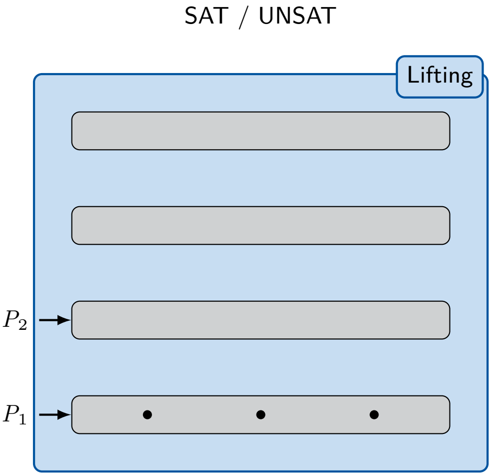

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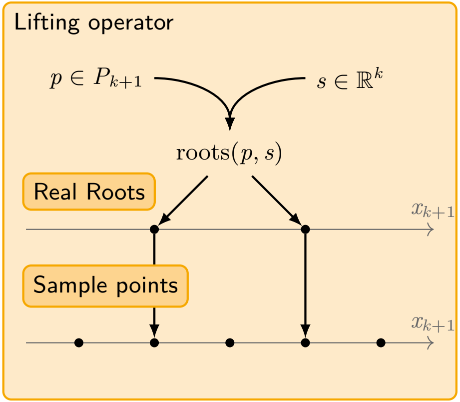

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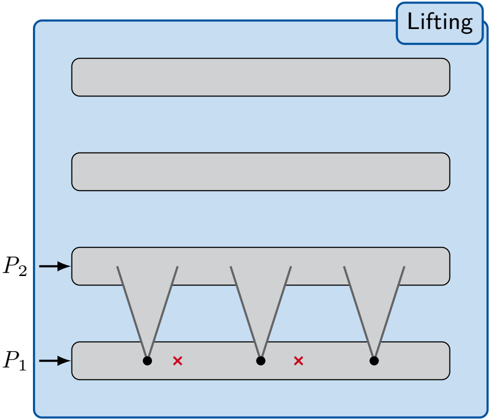

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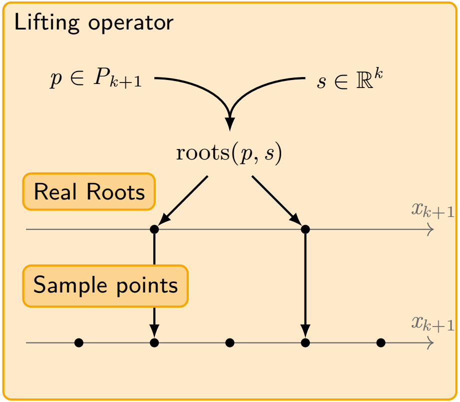

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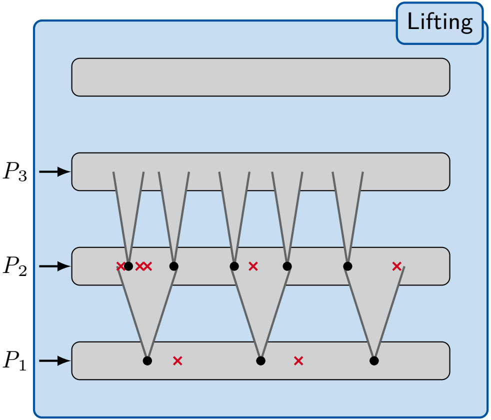

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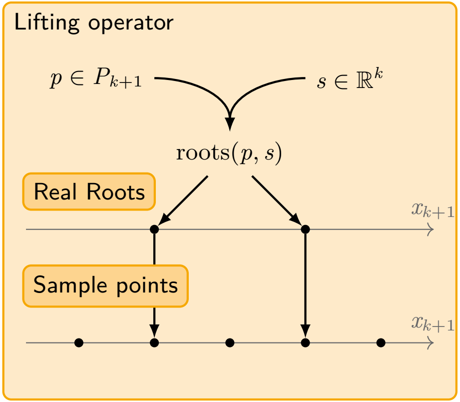

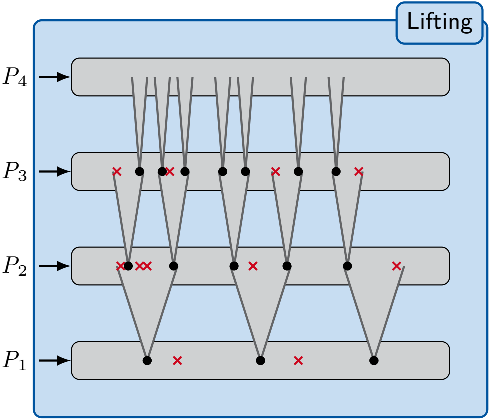

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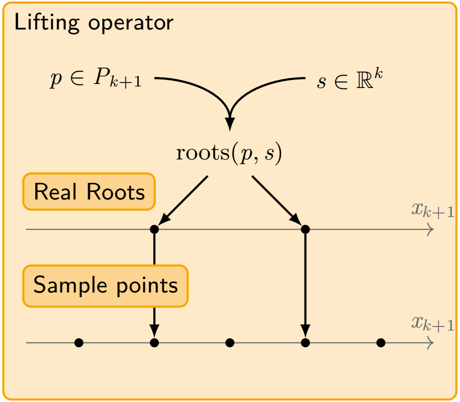

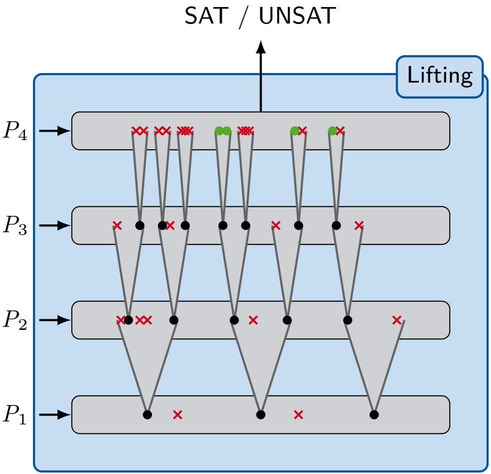

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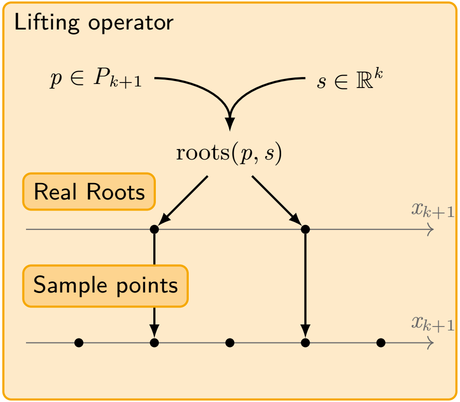

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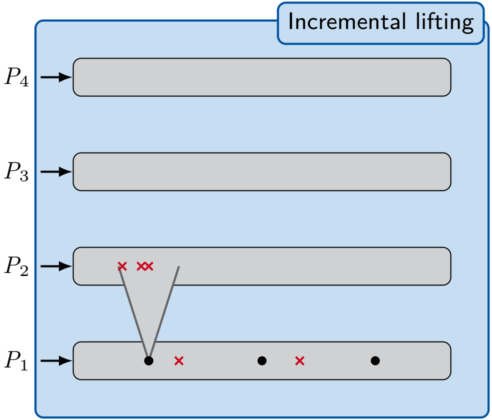

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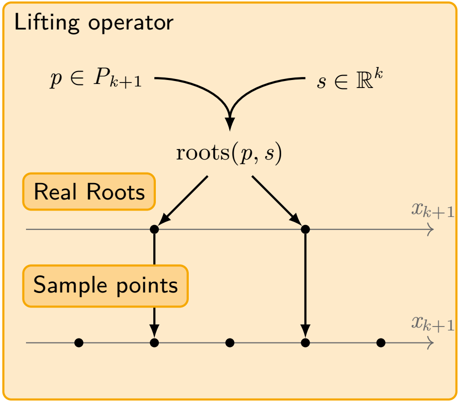

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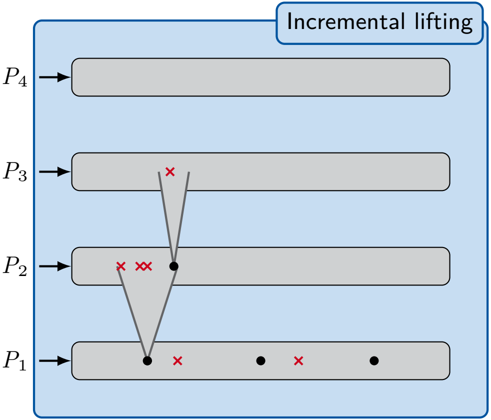

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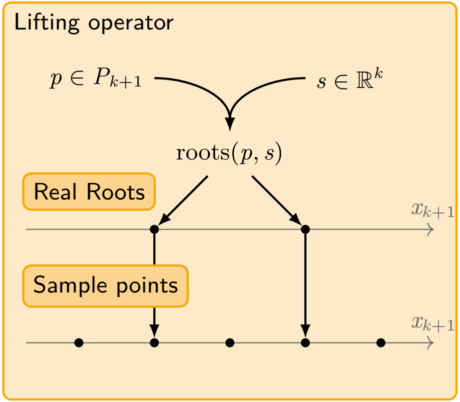

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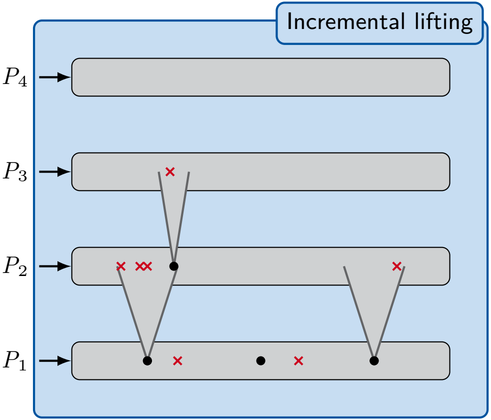

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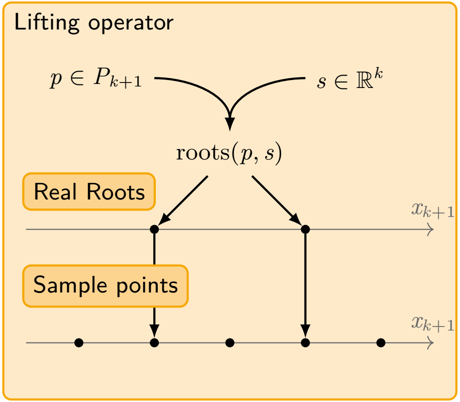

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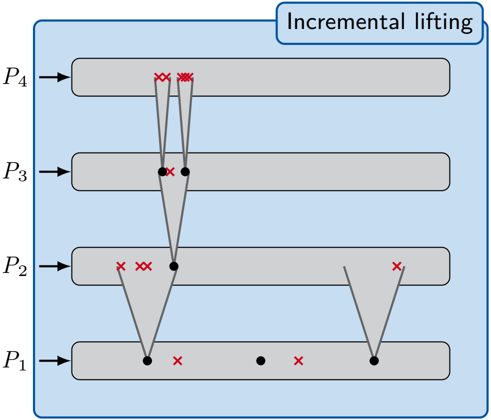

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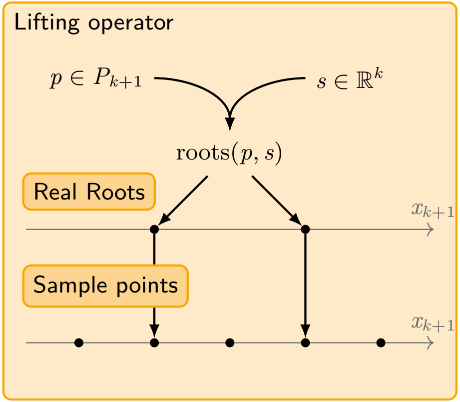
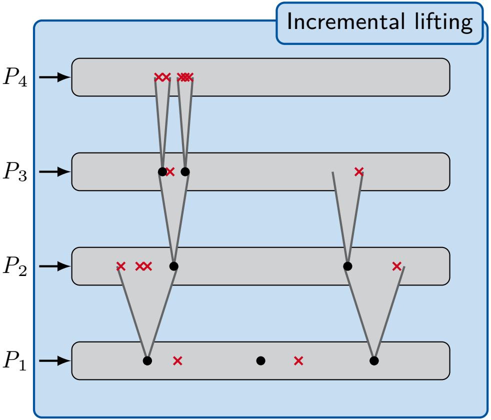


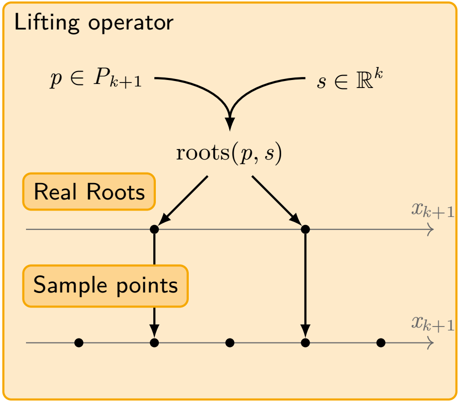
SAT / UNSAT



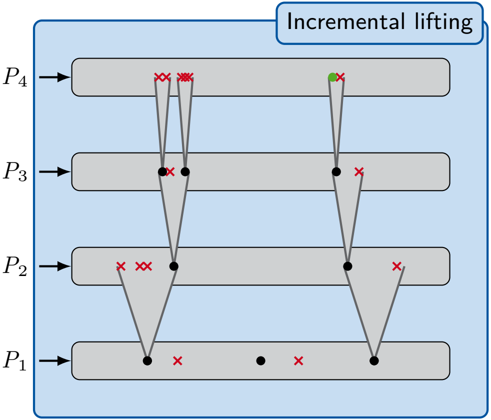
[Collins + 1991]

Incremental lifting

[JSC 2020]



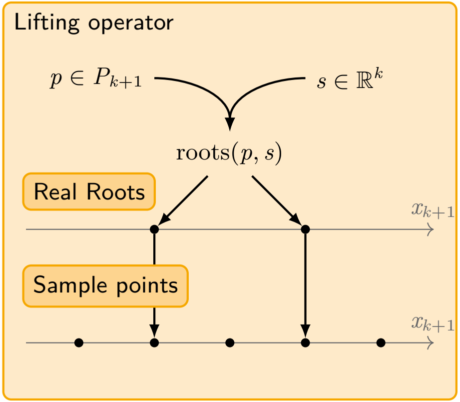
SAT / UNSAT

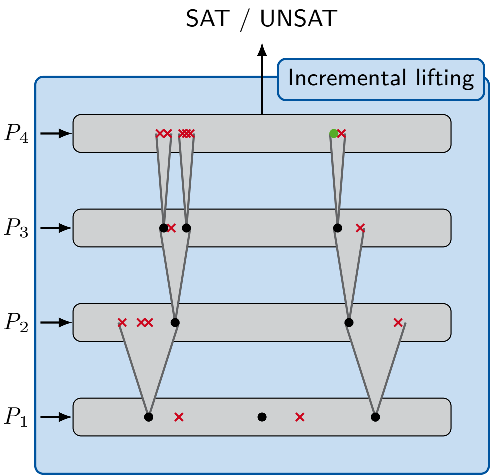


[Collins + 1991]

Incremental lifting

[JSC 2020]

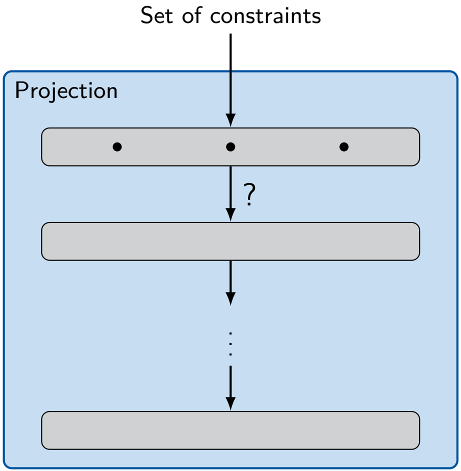




[Collins + 1991]

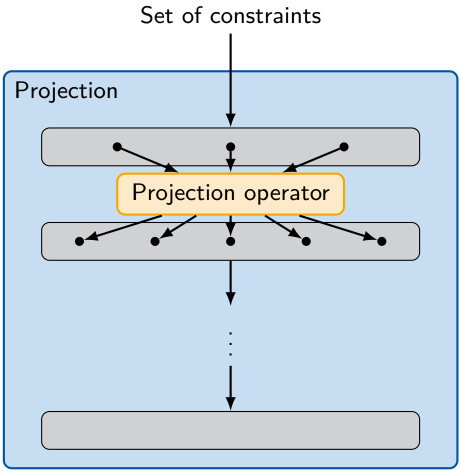
Incremental projection

[JSC 2020]



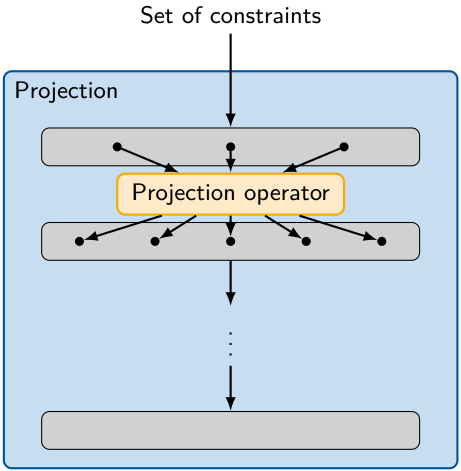
Incremental projection

[JSC 2020]



Incremental projection

[JSC 2020]

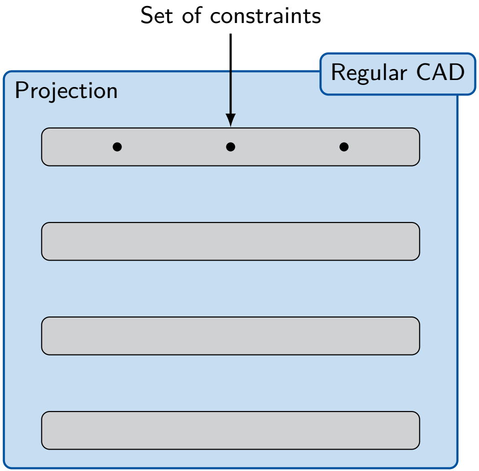


Projection operator

$$\begin{aligned}Proj(P) = &\{Proj(p) \mid p \in P\} \\ &\cup \{Proj(p, q) \mid p, q \in P\}\end{aligned}$$

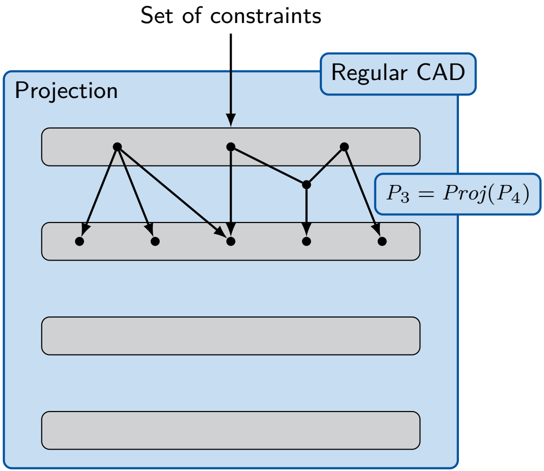
Incremental projection

[JSC 2020]



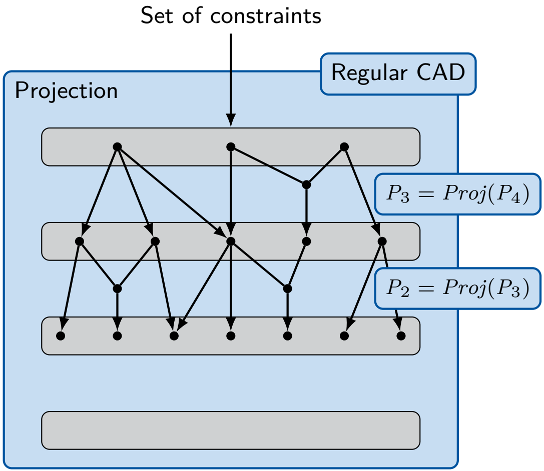
Incremental projection

[JSC 2020]



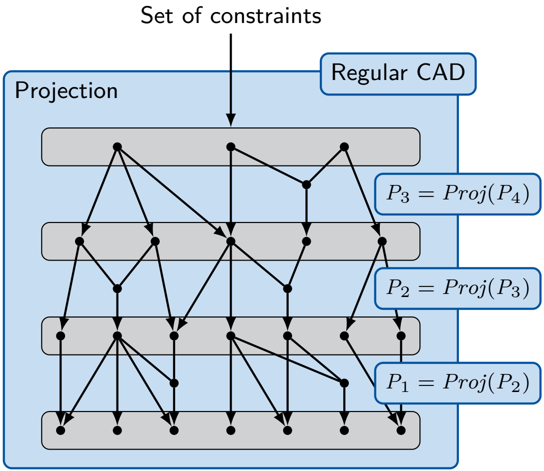
Incremental projection

[JSC 2020]



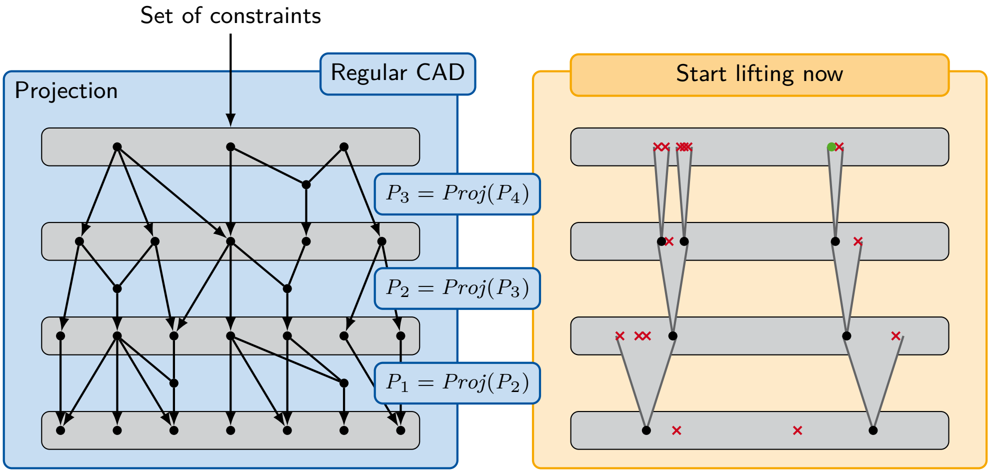
Incremental projection

[JSC 2020]



Incremental projection

[JSC 2020]



Incremental projection

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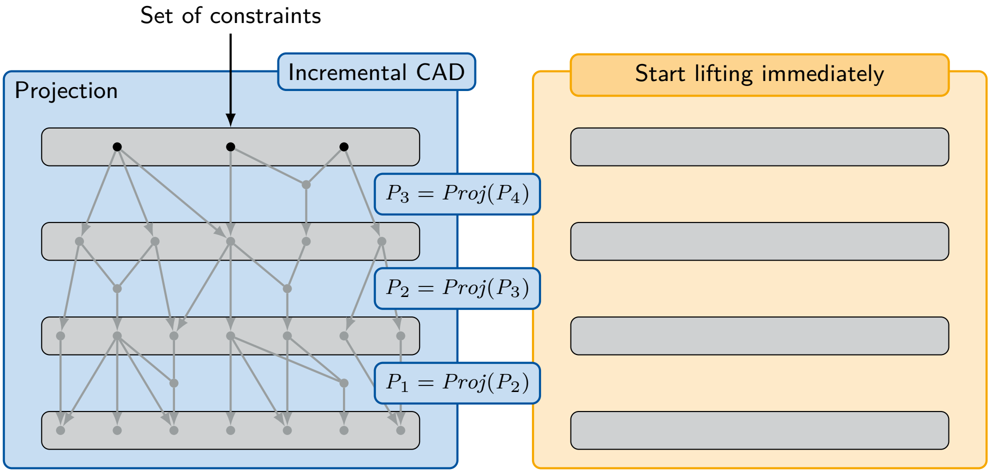

Incremental projection

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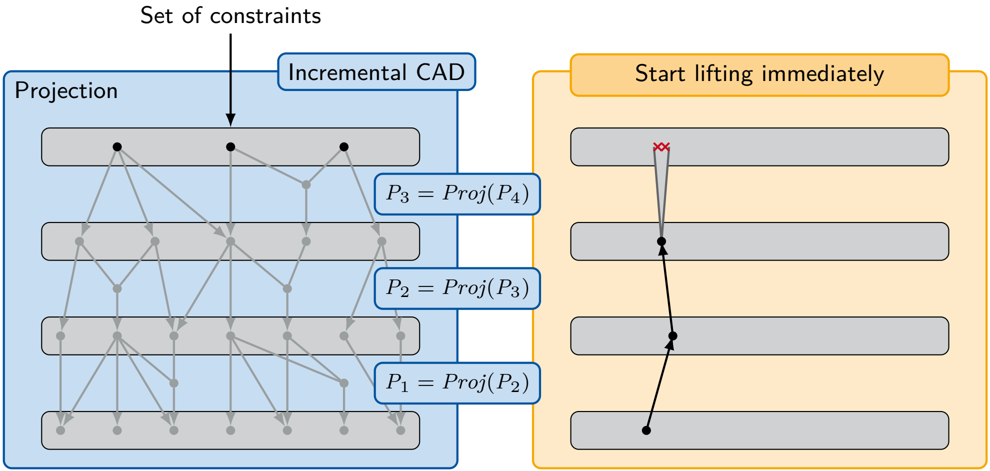

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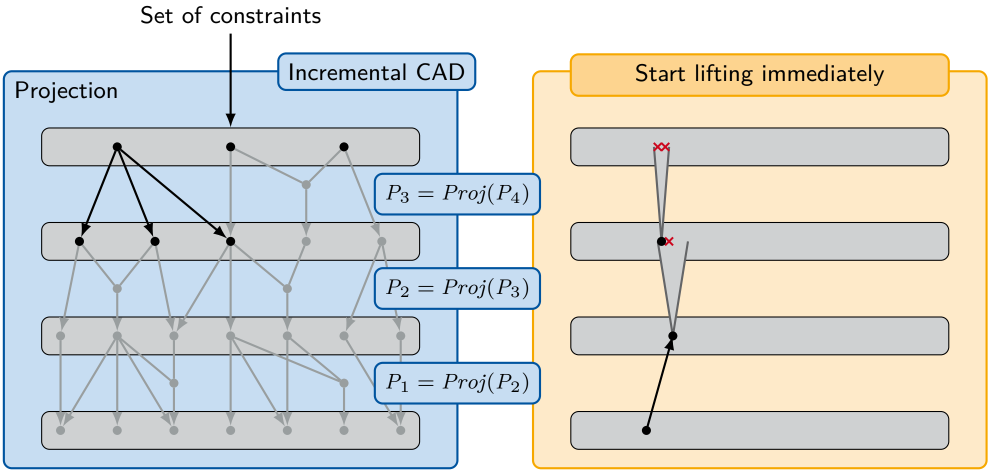

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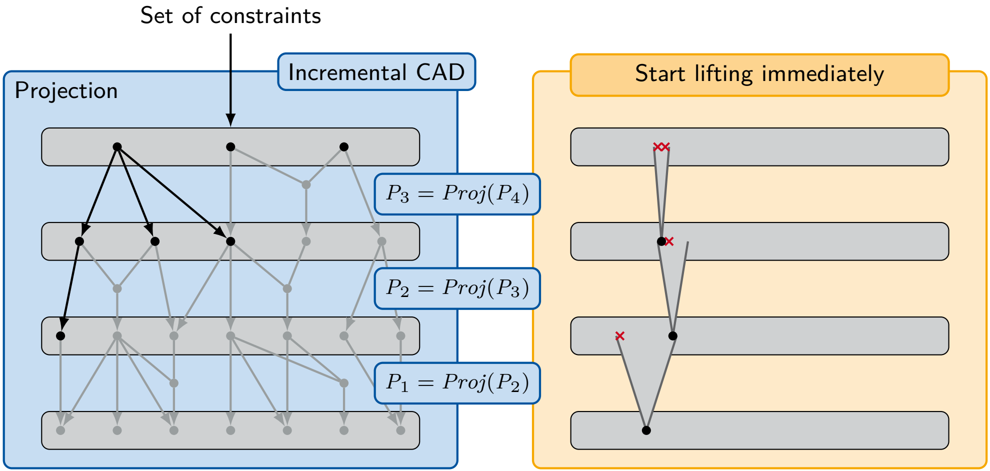

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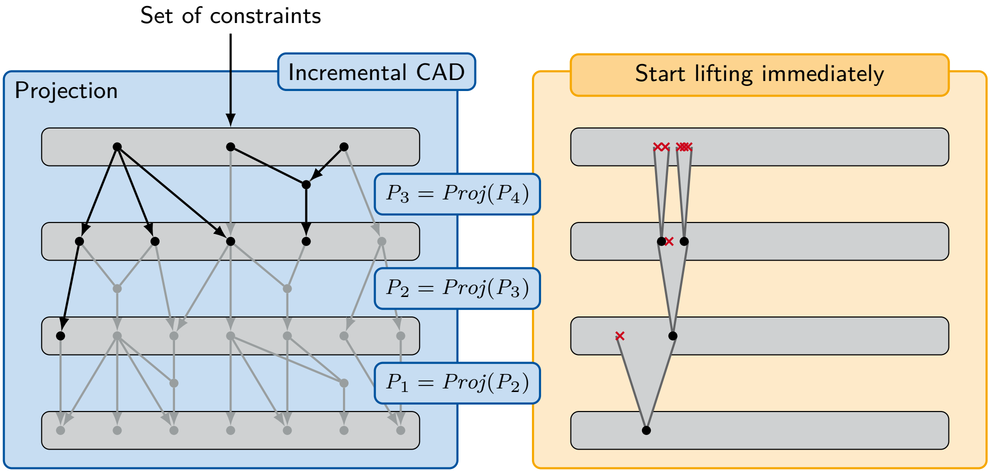
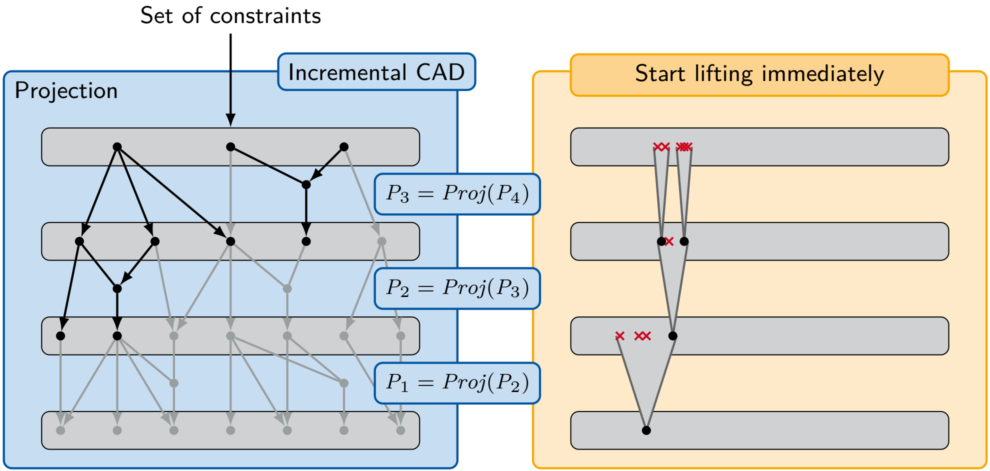


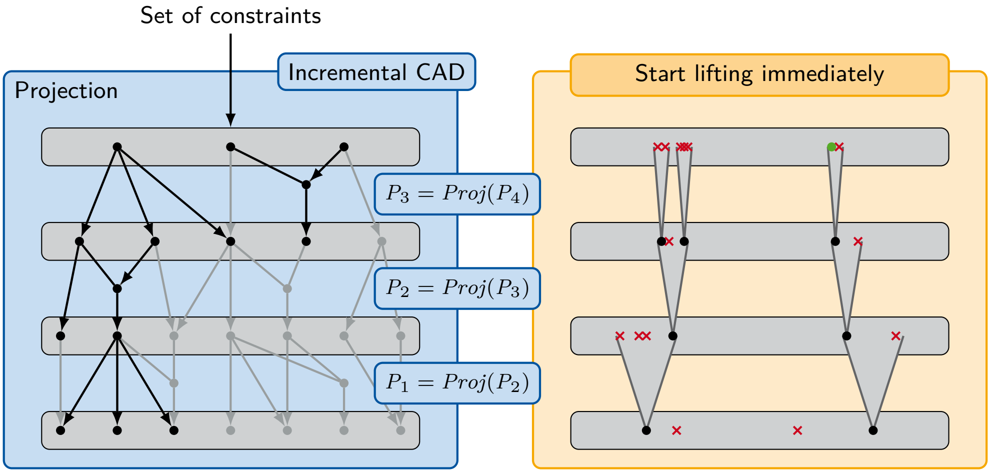
Incremental projection

[JSC 2020]



Incremental projection

[JSC 2020]



Heuristics & Additions

[JSC 2020]

- Level of incrementality
  Trade-off between potential gains and bookkeeping
- Projection operators
  Collins, Hong, McCallum, Lazard, Brown
  [SC^2 2017]
- Variable ordering
  [England + 2014] [Huang + 2014]
- Scheduling of projection and lifting
  Which polynomials and sample points to use first
- Equational constraints
  Exploits equalities to reduce projection
  [McCallum 1999] [SC^2 2018]
- Factorization of polynomials
  Ensures correctness, can reduce projection
- Minimal infeasible subsets
  Allow SAT solver to learn smaller conflicts
  [Jaroschek + 2015][Hentze 2017]

Experimental results

- Timeout: 120s
- Benchmarks: `QF_NRA` from SMT-LIB

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Solver	SAT	UNSAT	overall
`Naive`	4285	3496	7781
`None`	4293	3507	7800
`Simple`	4281	3889	8170
`Full`	4328	4250	8578
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- Incrementality is worth the effort
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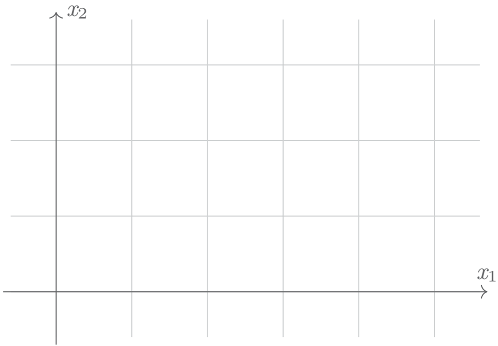
Projection operators

Solver	SAT	UNSAT	overall
`Collins`	4292	4150	8442
`Hong`	4301	4189	8490
`McCallum`	4320	4216	8536
`Lazard`	4322	4229	8551
`Brown`	4328	4250	8578

- Confirms conventional wisdom
- Margins may be surprising

Model-Constructing Satisfiability Calculus



[Jovanović + 2012] [Moura + 2013]

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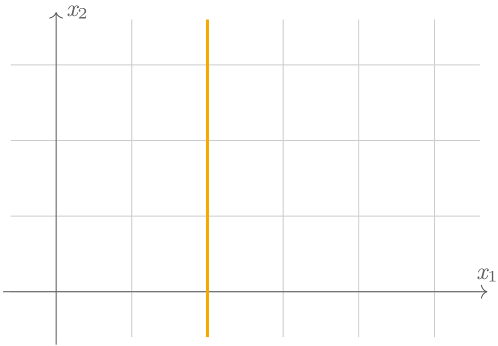

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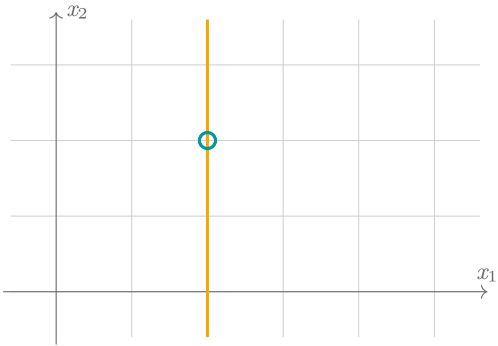

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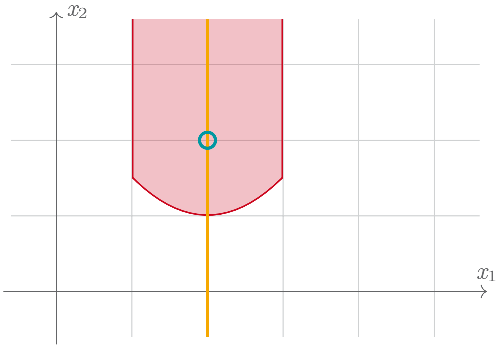

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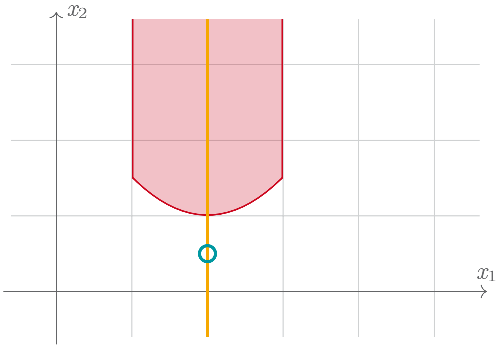

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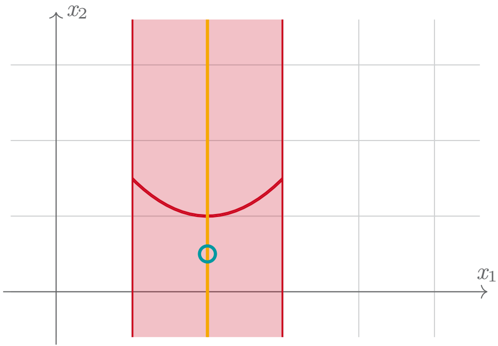

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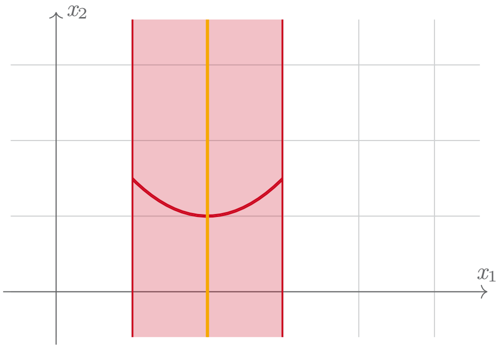

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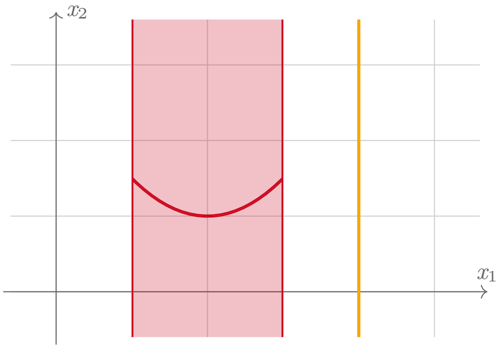

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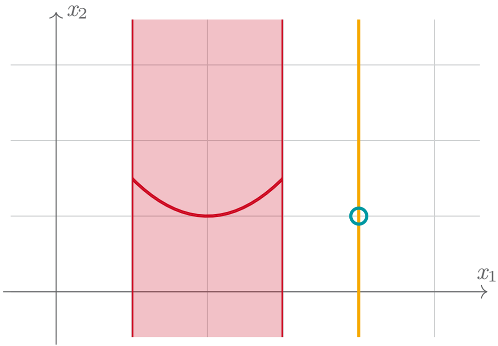

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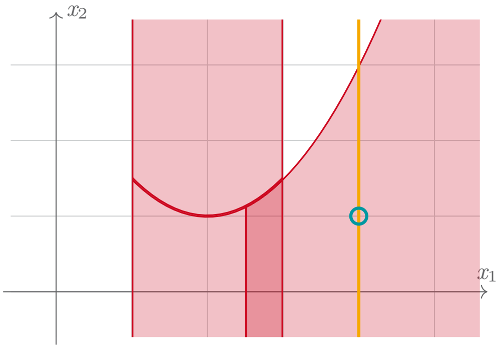

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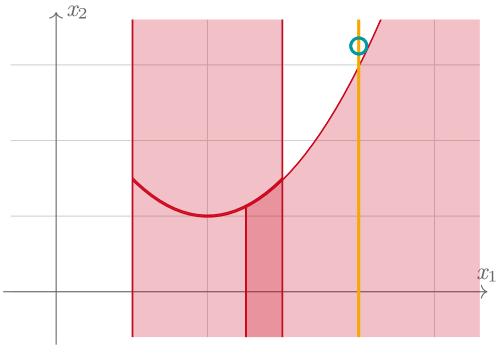

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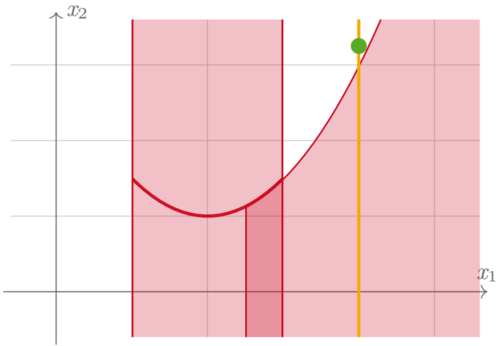

[Jovanović + 2012] [Moura + 2013]

Model-Constructing Satisfiability Calculus

(Lazy) SMT solver

[Jovanović + 2012] [Moura + 2013]

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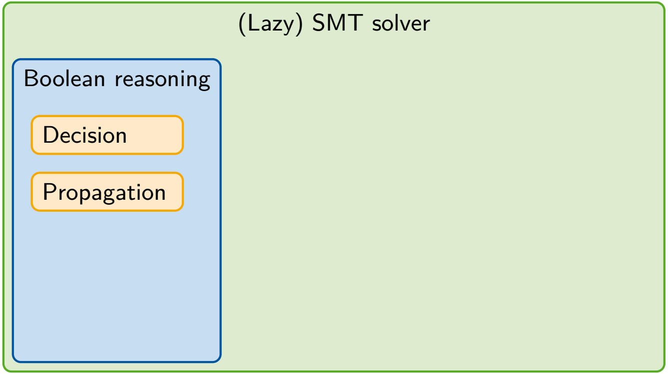

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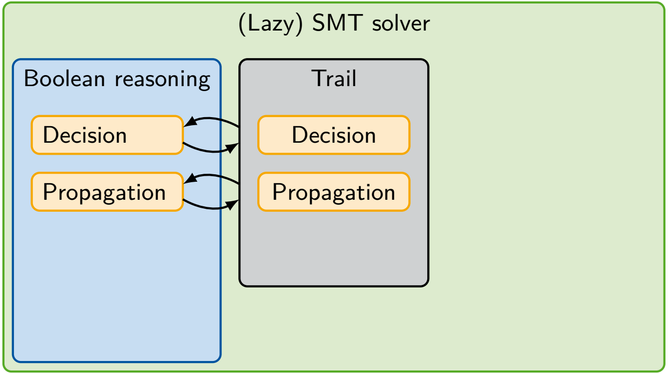

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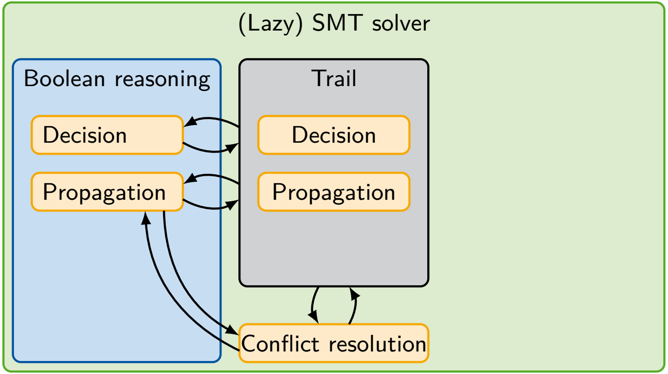

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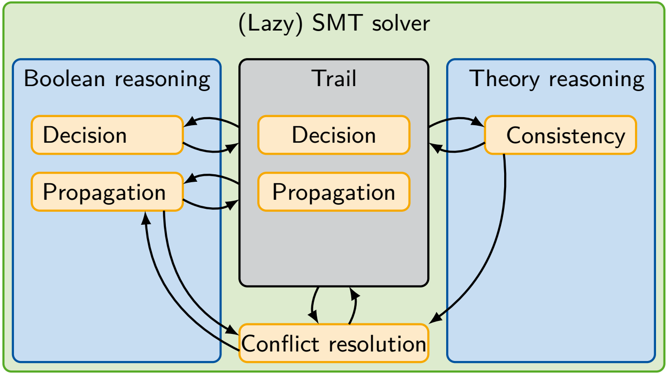

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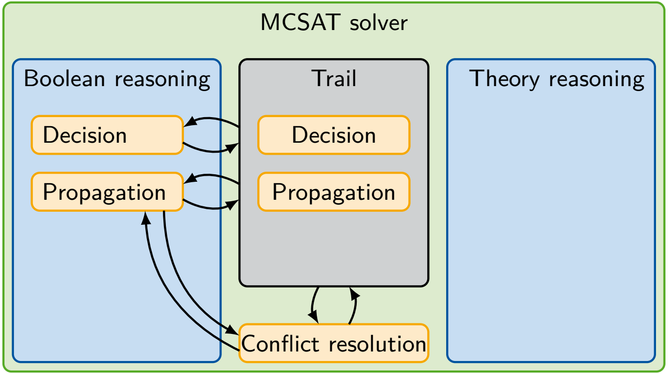

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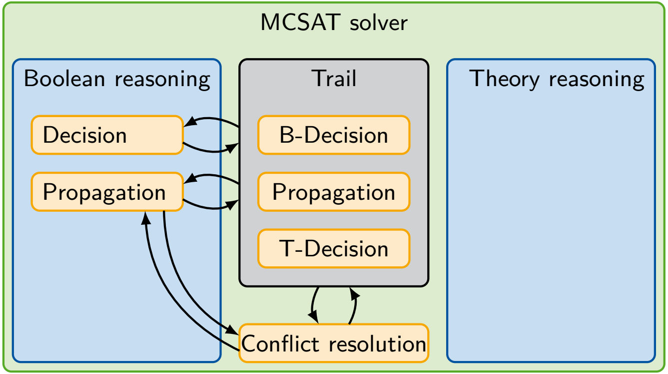

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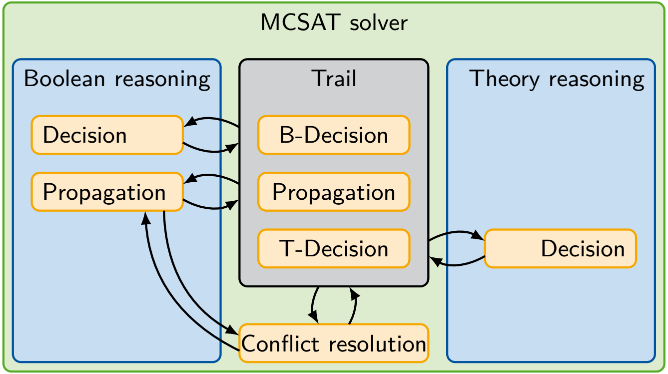

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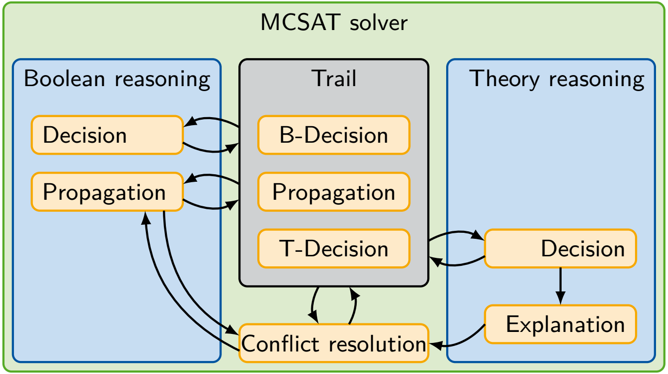

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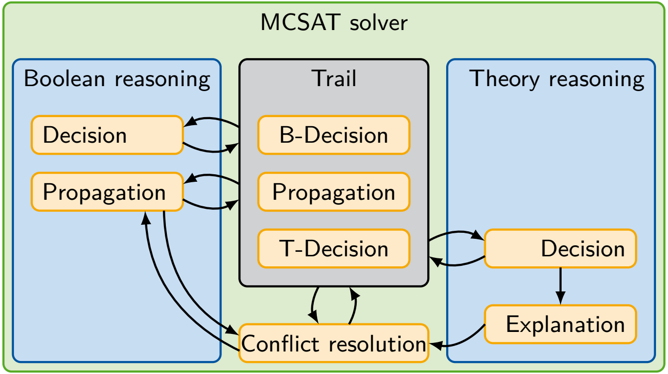
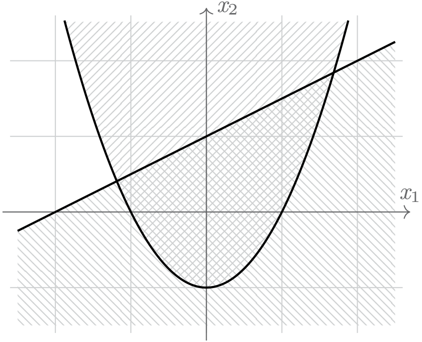


Trail: $[\![c_1, c_2]\!]$



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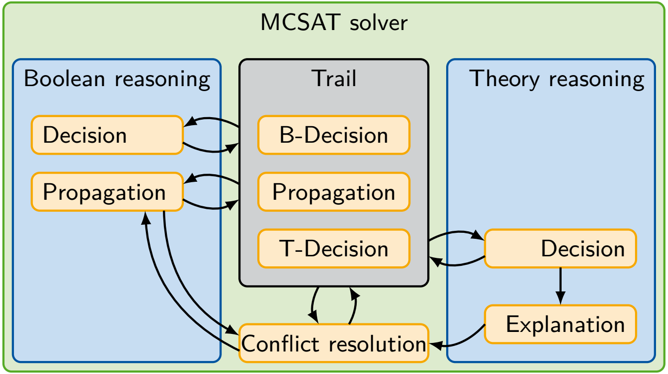
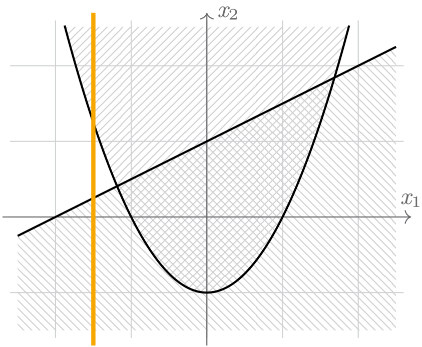


Guess: $x_1 \mapsto -1.5$

Trail: $[\![c_1, c_2, x_1 \mapsto -1.5]\!]$



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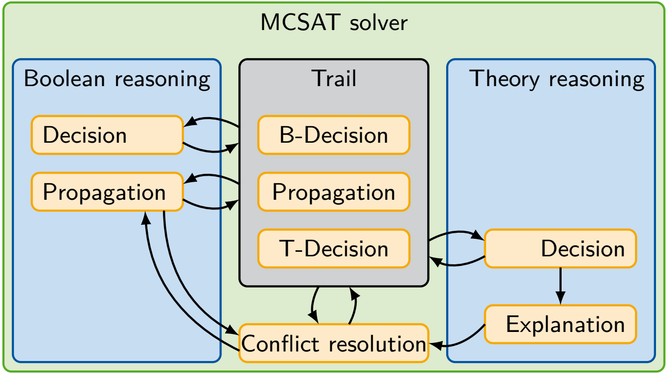
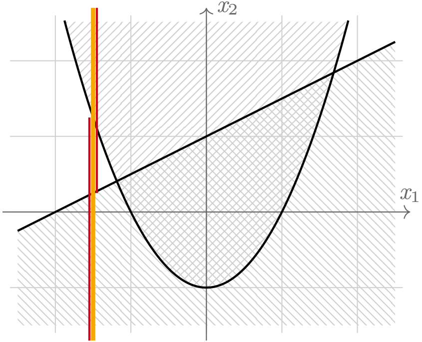


No assignment for x_2. Reason: c_1, c_2

Trail: $[\![c_1, c_2, x_1 \mapsto -1.5]\!]$



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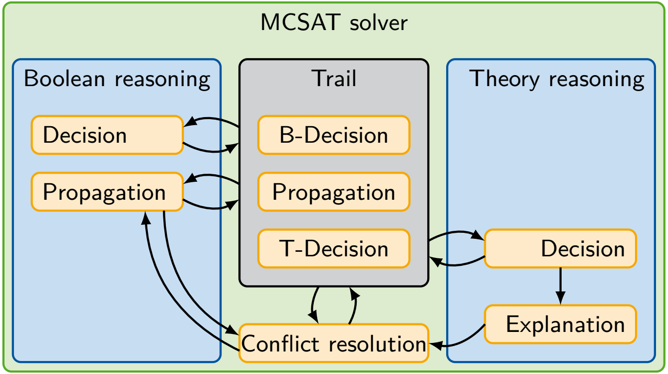

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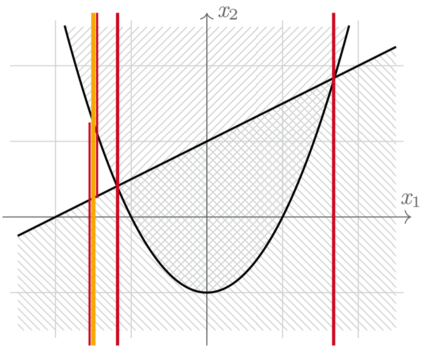

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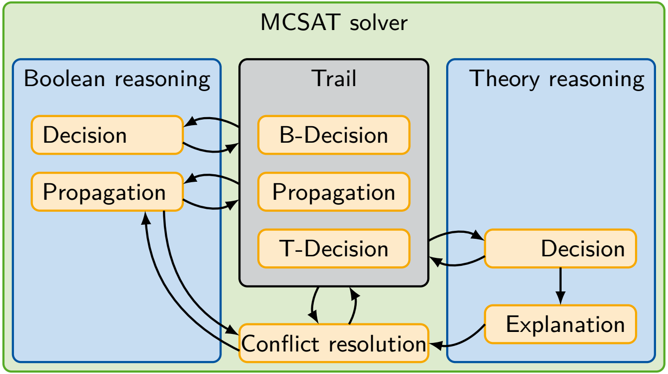

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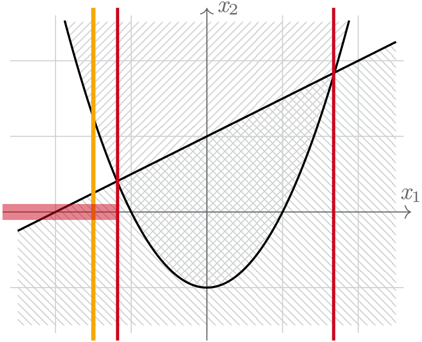

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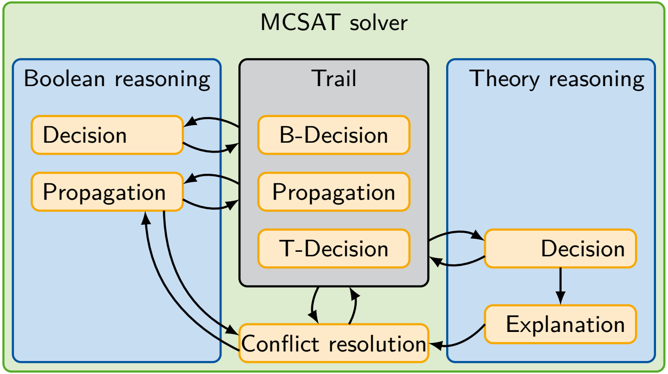
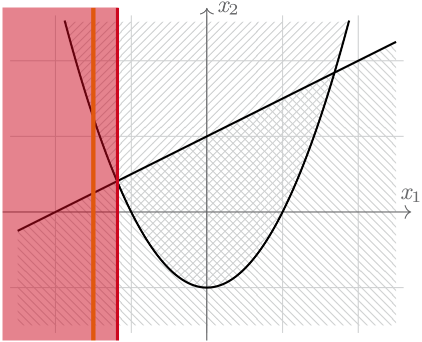


$(c_1 \wedge c_2) \rightarrow (\alpha_2 \leq x_1)$

Trail: $[\![c_1, c_2]\!]$



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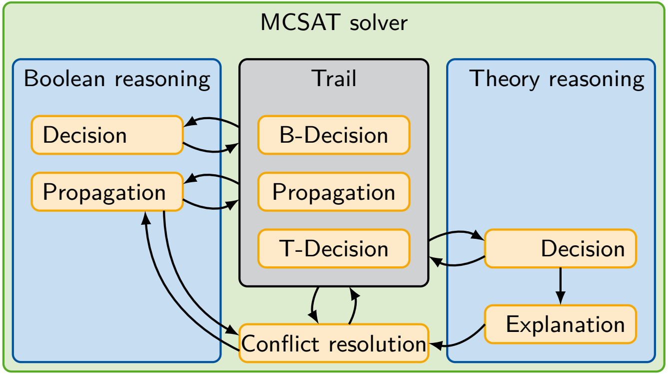
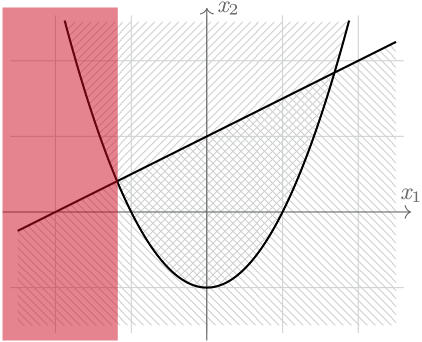
Model-Constructing Satisfiability Calculus



Conflict resolution

Trail: $[\![c_1, c_2]\!]$



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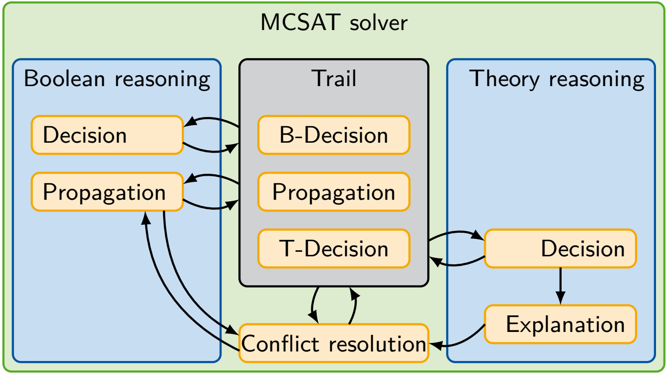
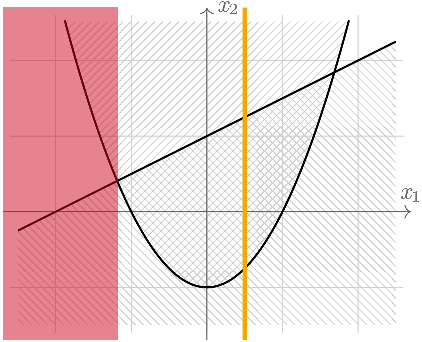
Model-Constructing Satisfiability Calculus



Guess: $x_1 \mapsto 0.5$

Trail: $[\![c_1, c_2, x_1 \mapsto 0.5]\!]$



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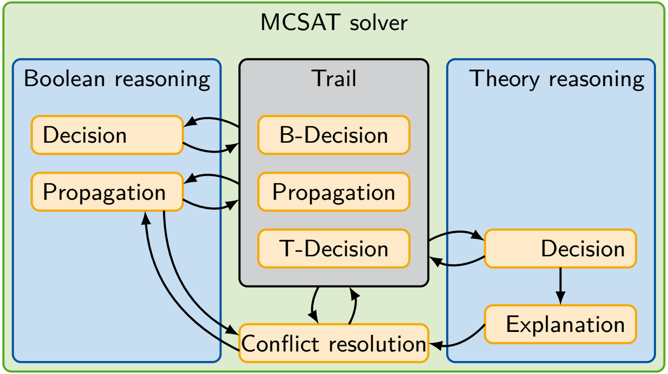
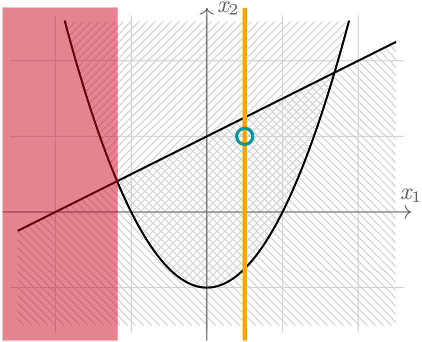


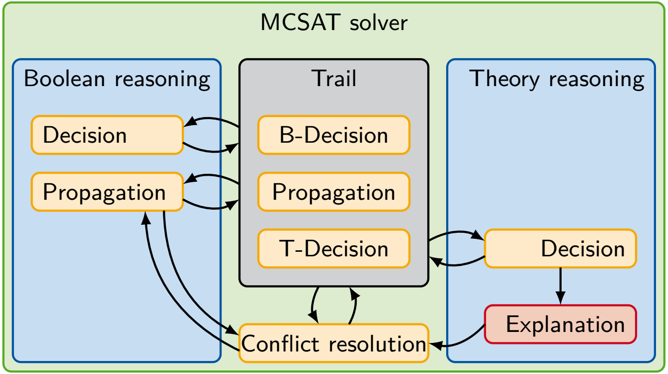
Guess: $x_2 \mapsto 1$

Trail: $[\![c_1, c_2, x_1 \mapsto 0.5, x_2 \mapsto 1]\!]$



[Jovanović + 2012] [Moura + 2013]

Model-Constructing Satisfiability Calculus



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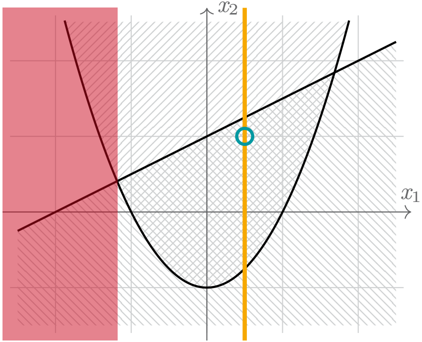

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Explanation functions

Definition (Explanation function)

Let $\mathcal{A} = \{x_1 \mapsto \alpha_1, \dots, x_k \mapsto \alpha_k\}$ and $C = \{c_1, \dots\}$ constraints over $x_1, \dots x_{k+1}$.
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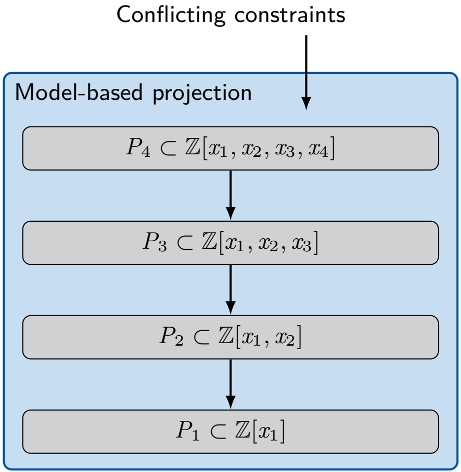
Model-based CAD [Jovanović + 2012], Fourier-Motzkin [Jovanović + 2013]

CAD-based / NLSAT

Conflicting constraints

[Jovanović + 2012]

CAD-based / NLSAT



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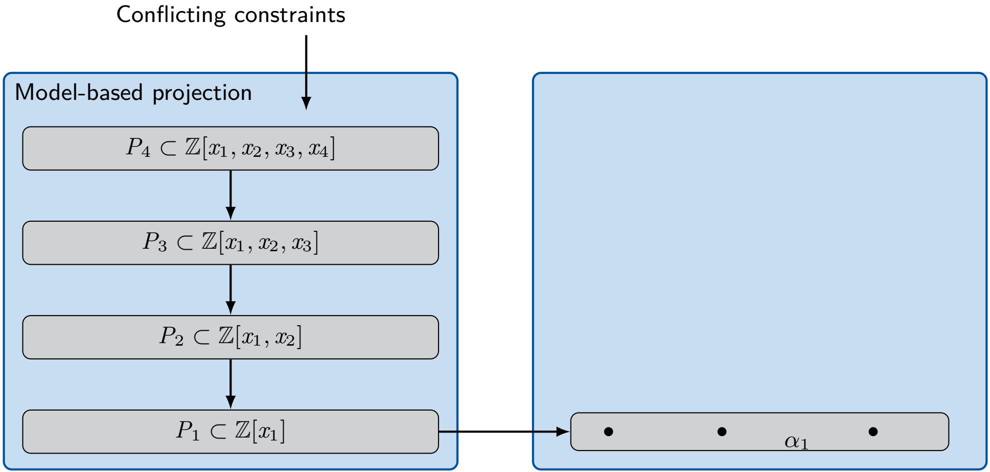

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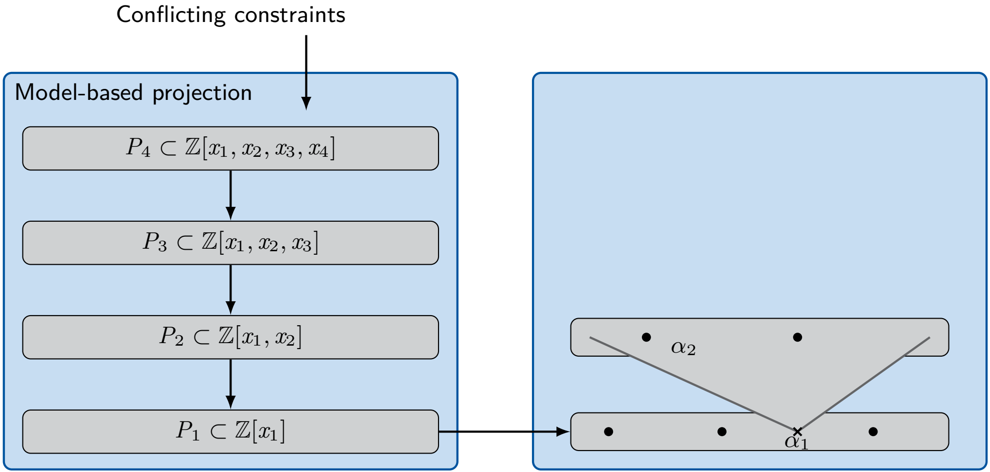

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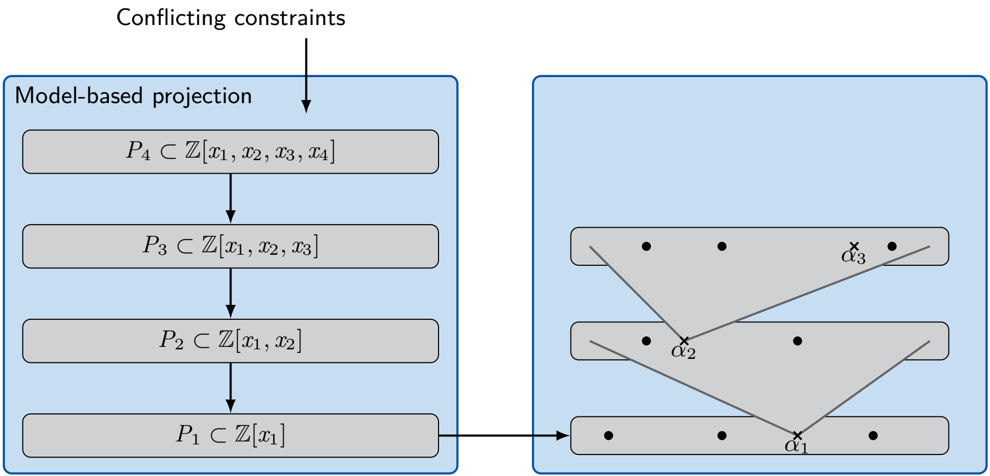

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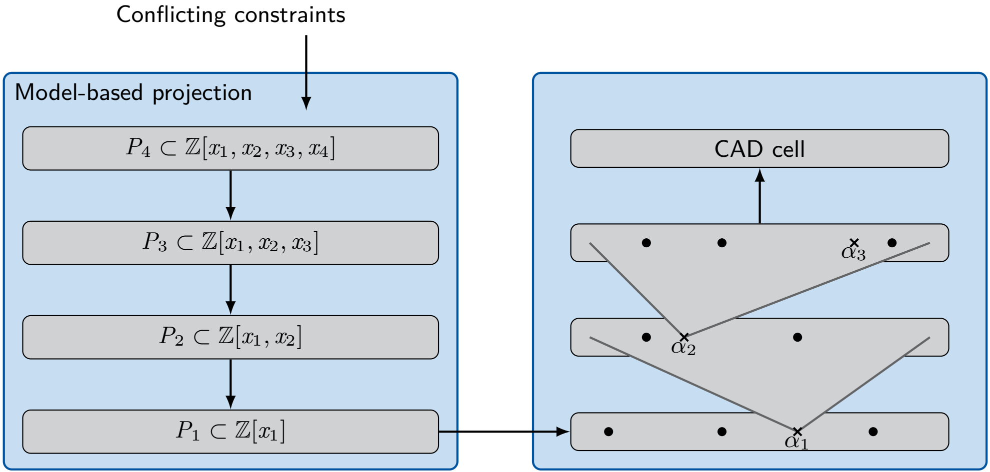

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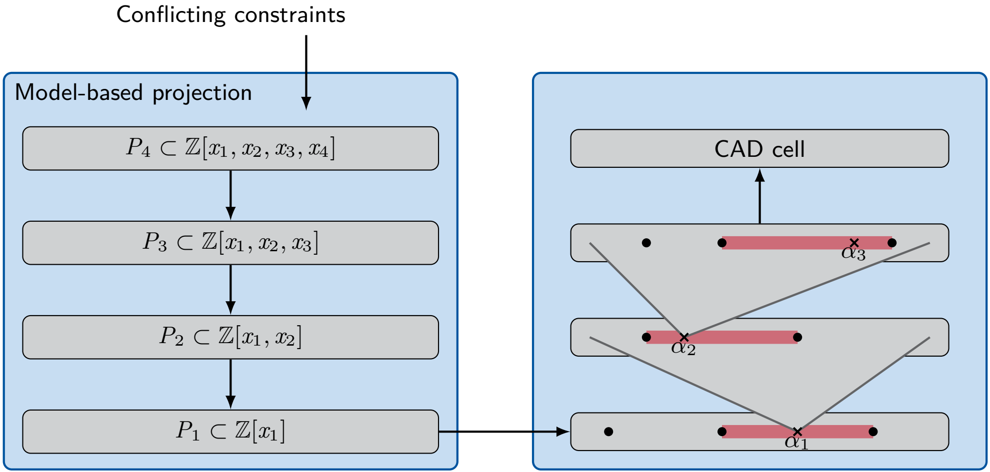

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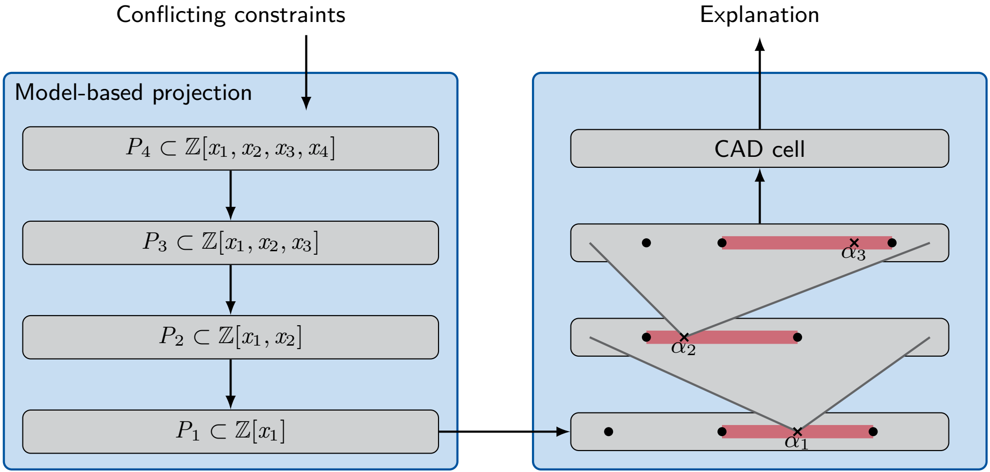

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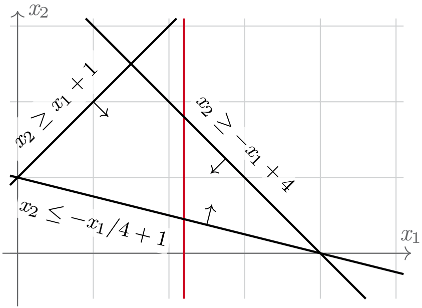

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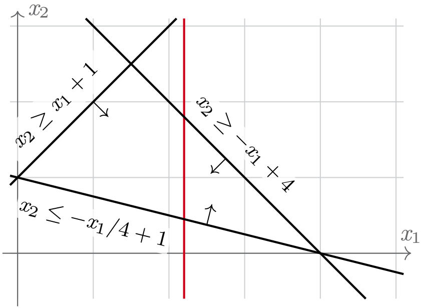

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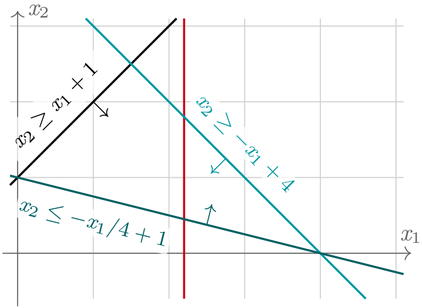
Conflicting constraints

↓

Identify lower and upper bounds

$-x_1 + 4 \leq \quad x_2 \quad \leq -x_1/4 + 1$



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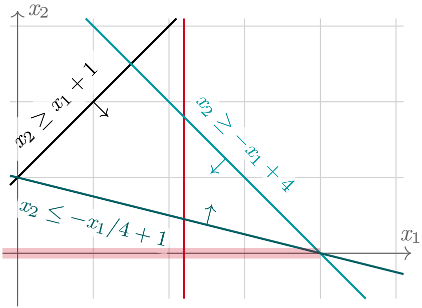

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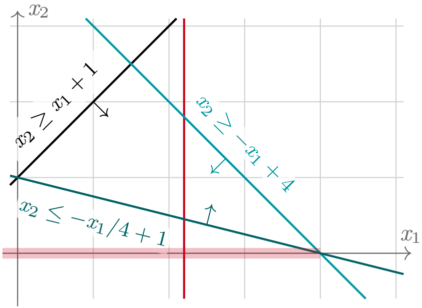
Explanation



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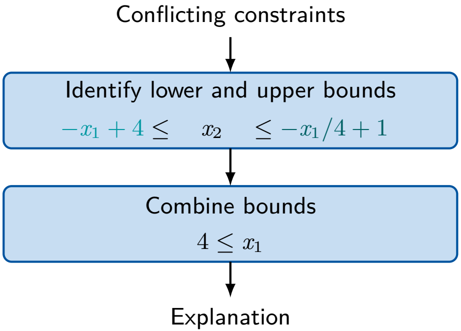

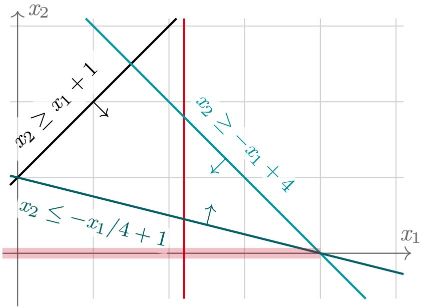

- Strict inequalities $\quad 2x_1 + 1 < x_2 < x_1 + 2 \quad x_1 \mapsto 2$
- Disequalities $\quad 2x_1 + 1 \leq x_2 \leq x_1 + 2 \land x_2 \neq 3 \quad x_1 \mapsto 1$
- Nonlinear coefficients $\quad x_1^2 + 3 \leq x_2 \leq 2x_1 \quad x_1 \mapsto 1$

[Jovanović + 2013]

Other explanation functions

Virtual cubstitution [SC^2 2017]

- Nonlinear but low-degree
- Flexible dimensionality of generated cells

[Brown 2013] [Brown + 2015]

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Composition of explanation functions

- `SMT-RAT`: modular composition
- Sequential & parallel

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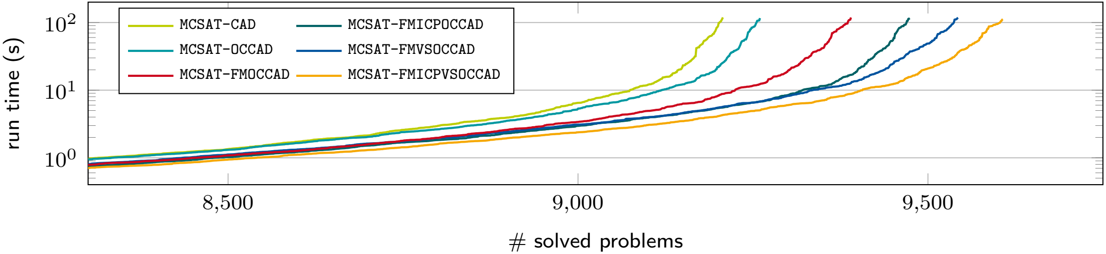

[Brown 2013] [Brown + 2015]

Theory variable ordering

[SC^2 2019]

- Well-known: variable ordering is crucial for CAD, VS, FM, ...
- Usually: static ordering for theory reasoning
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- Usually: static ordering for theory reasoning
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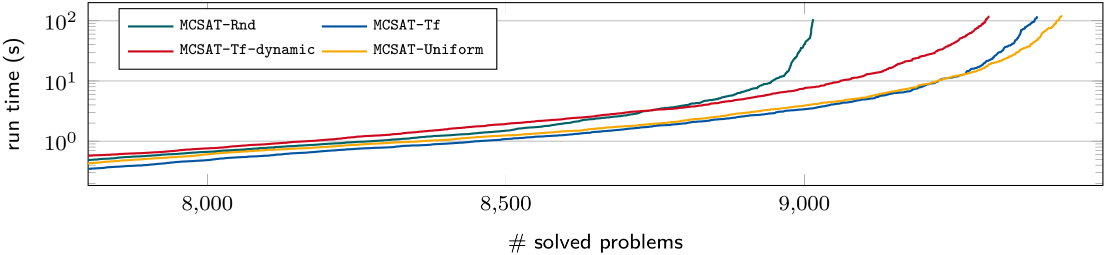
- Observation: `MCSAT-Tf` and `MCSAT-Uniform` perform best
- But: `MCSAT-Tf-dynamic` is worse



Comparison of lazy SMT and MCSAT

[SC² 2019]

- Two similar proof systems: lazy SMT and MCSAT
- Experience: MCSAT better for nonlinear
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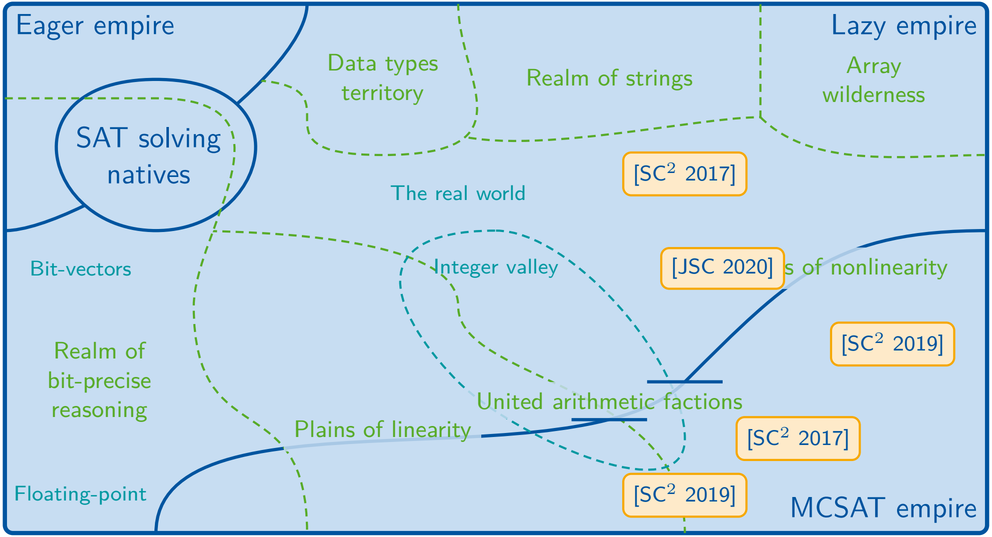
Theorem

Lazy SMT and MCSAT are algorithmically equivalent.*

*: terms and conditions apply

Summary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Data types territory
Realm of strings
Array wilderness
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The real world
[SC² 2017]
Bit-vectors
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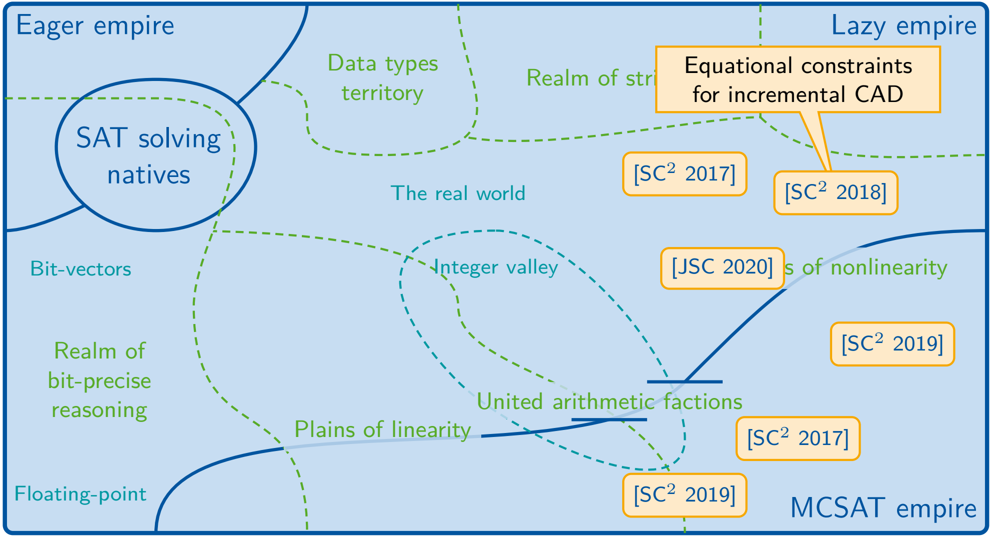

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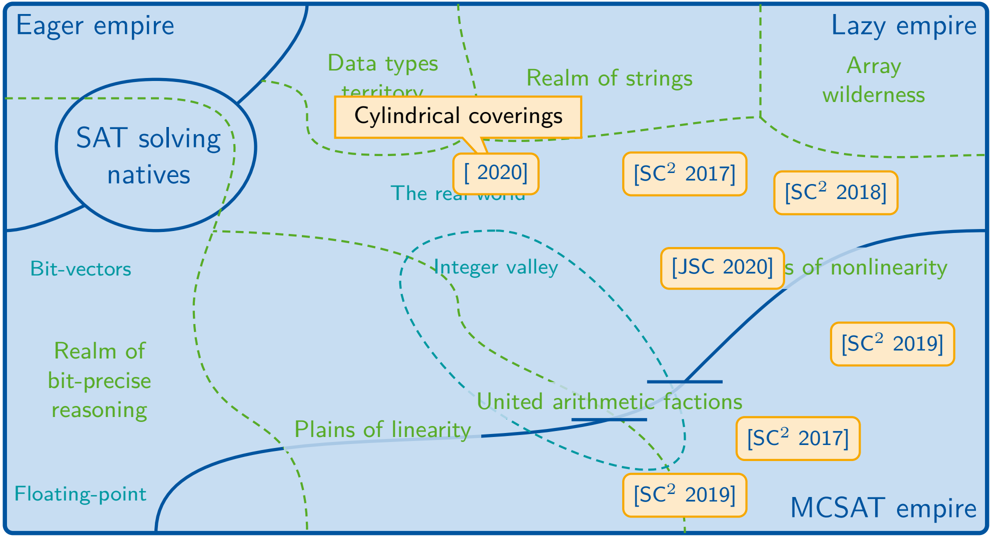
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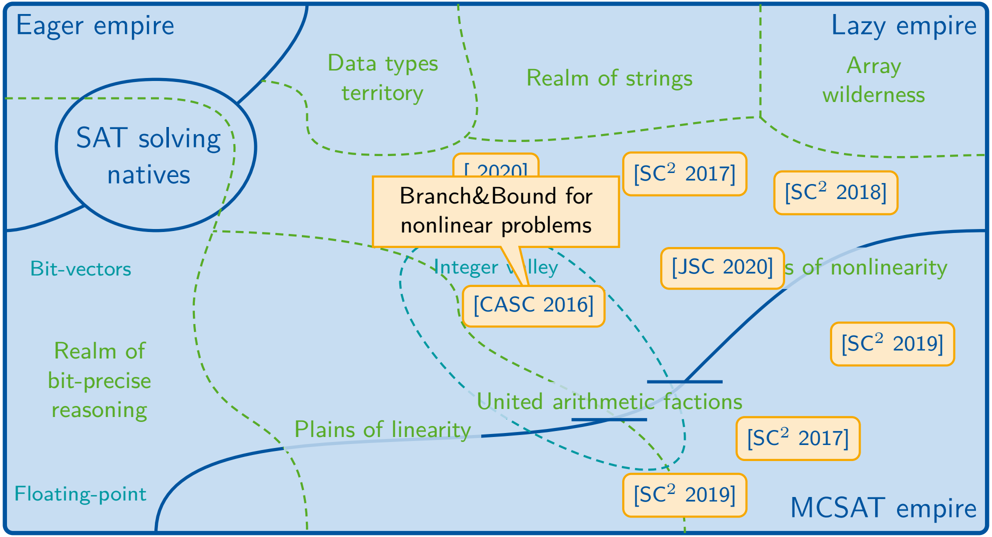

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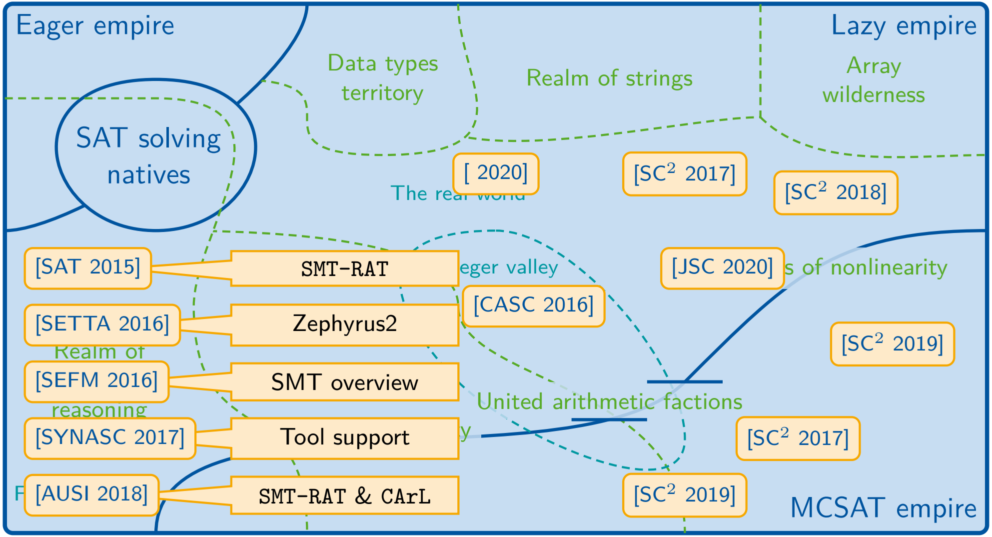
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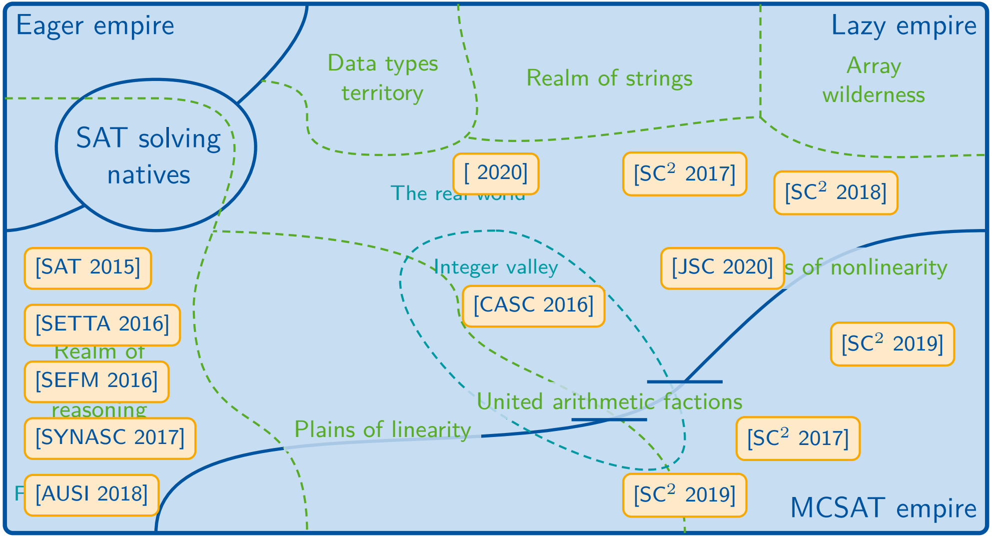
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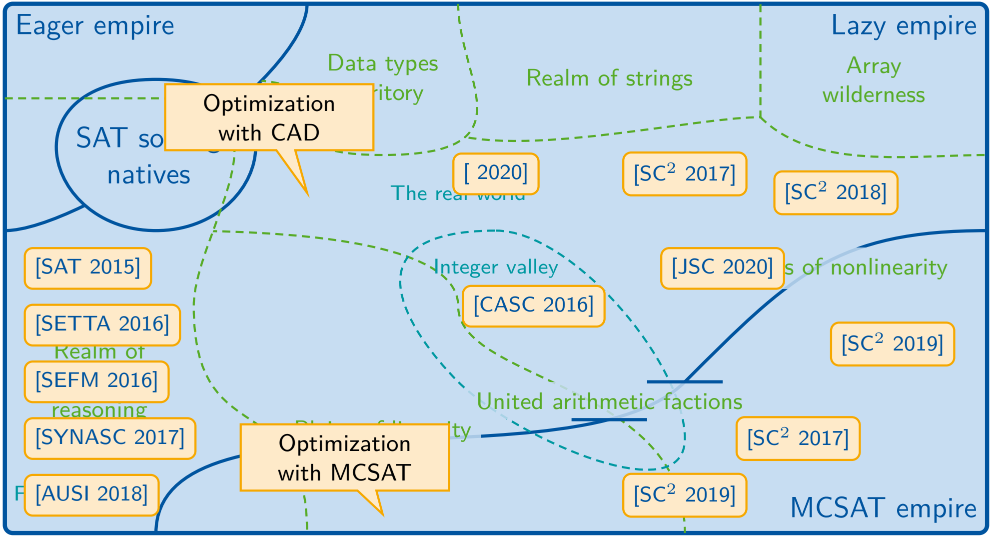
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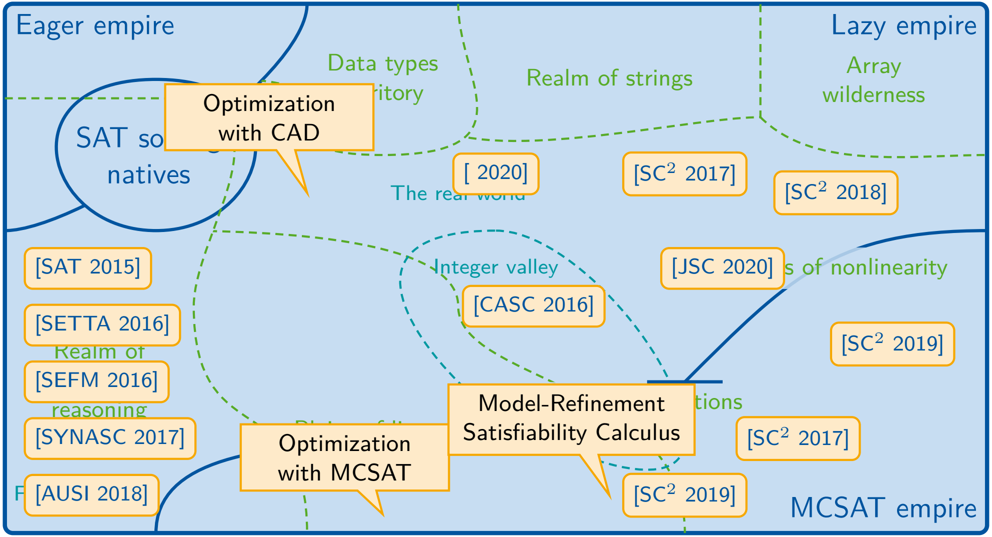
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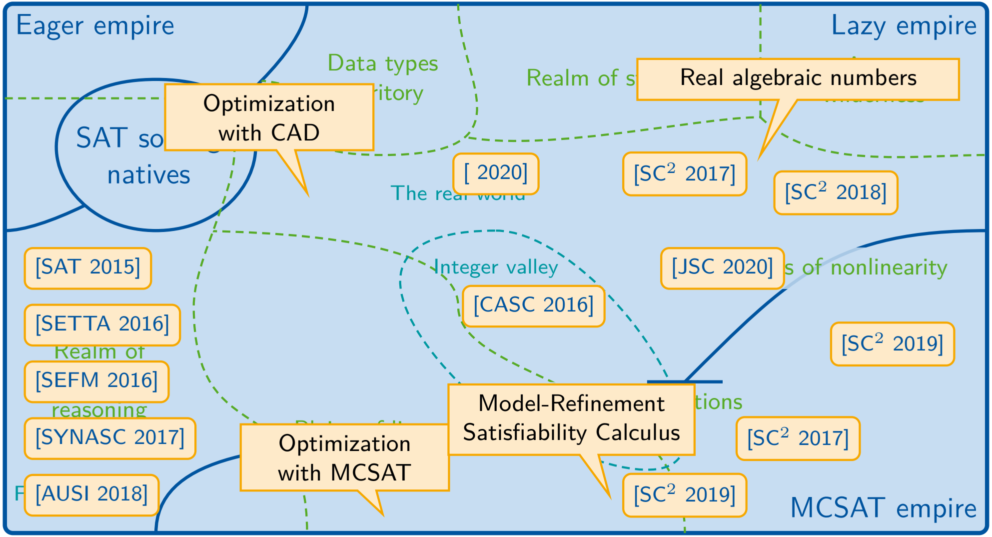
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Directions for future research:

- Optimize CAD integration: better scheduling, NuCAD, parallelization, ...
- Novel adaptations of CAD: CAC [2020], NuCAD [Brown 2015], ...
- New methods for MCSAT: explanations, orderings, assignment finders, ...
- More applications: optimization, dedicated heuristics, ...

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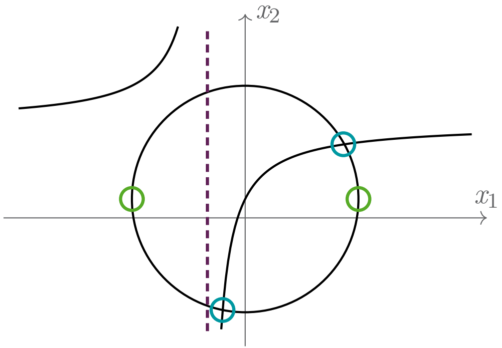
CAD projection

Example: McCallum

$$Proj(P) = \{\mathrm{coeffs}(p), \mathrm{disc}(p) \mid p \in P\} \cup \{\mathrm{res}(p, q) \mid p, q \in P\}$$



[McCallum 1985]

Minimal Infeasible Subsets

[Hentze 2017]

- Set of constraints $\{c_1, c_2, \ldots\}$ is infeasible

[Jaroschek + 2015] [Chvátal 1979]

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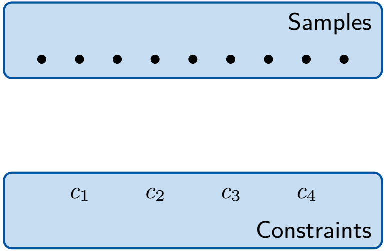

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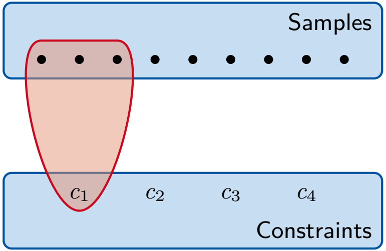

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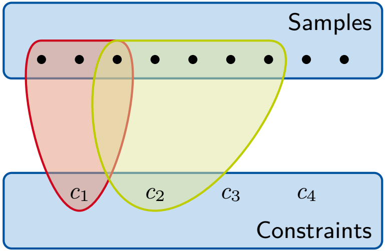

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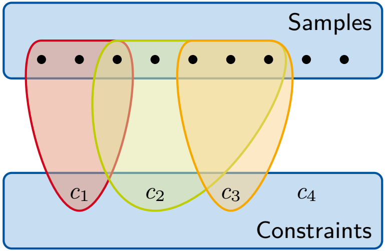

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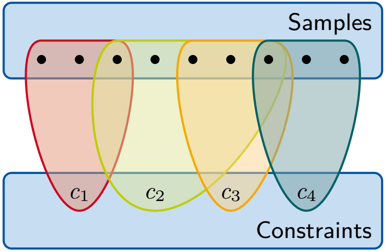

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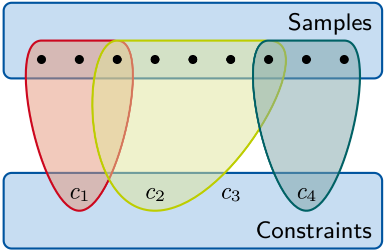

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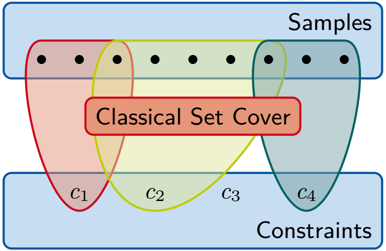

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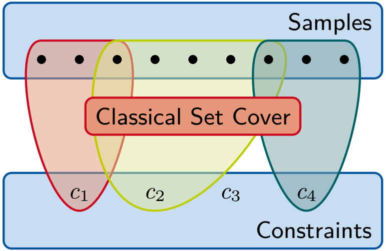

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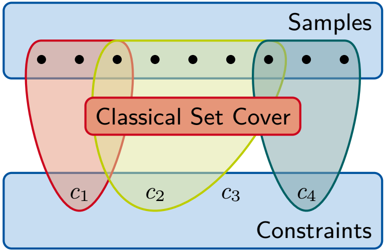
- Significant improvements for MIS ...
- but no effect on solver performance



Solver	SAT	UNSAT	overall
`Greedy-PP`	4327	4249	8576
`Greedy-Weighted`	4329	4247	8576
`Hybrid`	4328	4248	8576
`Trivial`	4325	4251	8576
`Exact`	4328	4249	8577
`Greedy`	4328	4250	8578

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Using CAD for ...

...Quantifier Elimination [Neuhäuser 2018]

$$(\exists x.\ \forall y.\ \varphi(x, y, z)) \Leftrightarrow \varphi'(z)$$

- Validation of our CAD implementation
- Show reuse of implementation
- Results comparable to `QEPCAD B`

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...Integer problems [CASC 2016]

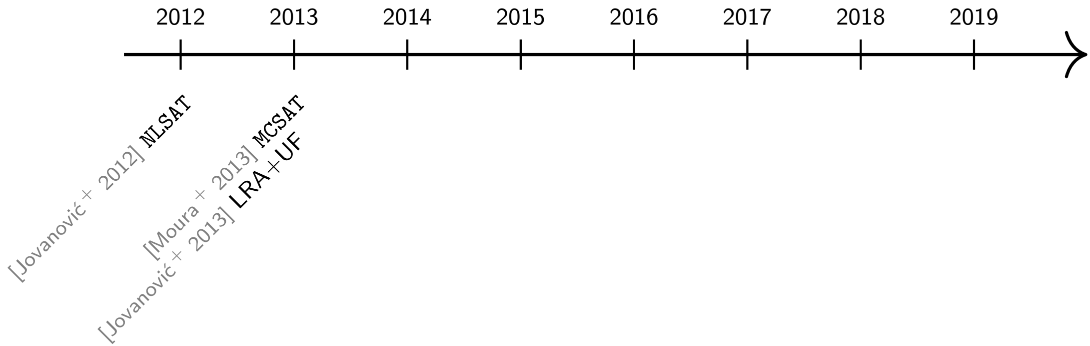
$$\exists x.\ \varphi(x) \qquad x \in \mathbb{Z}$$

- Common approach: bit-blasting
  mostly aims for small domains
- Our approach: Branch & Bound
  mostly aims for algebraically easy problems & UNSAT

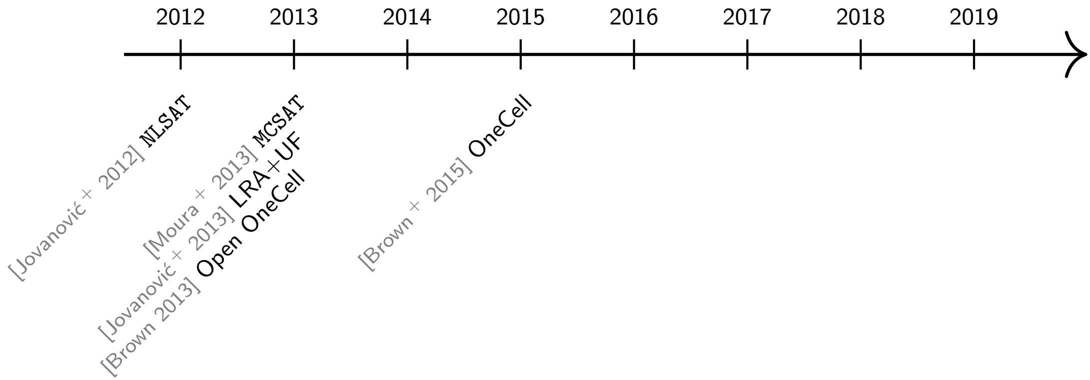
Solver	SAT	UNSAT	overall
`NIA-B&B`	1044	518	1562
`NIA-Blast`	4367	26	4393
`NIA-Full`	4490	508	4998

[Brown 2003] [Fuhs + 2007]

MCSAT history



MCSAT history



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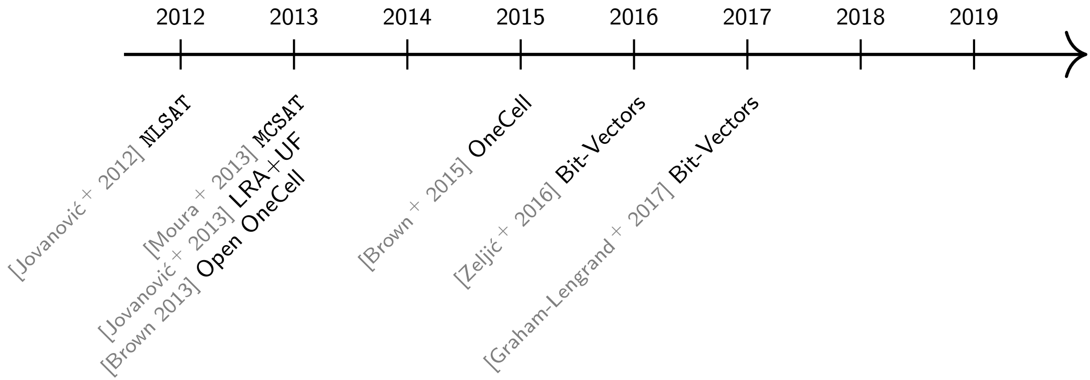

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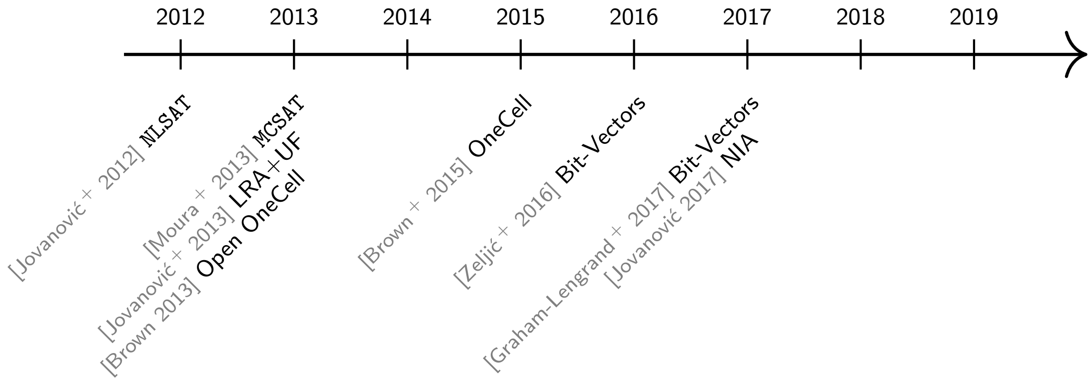

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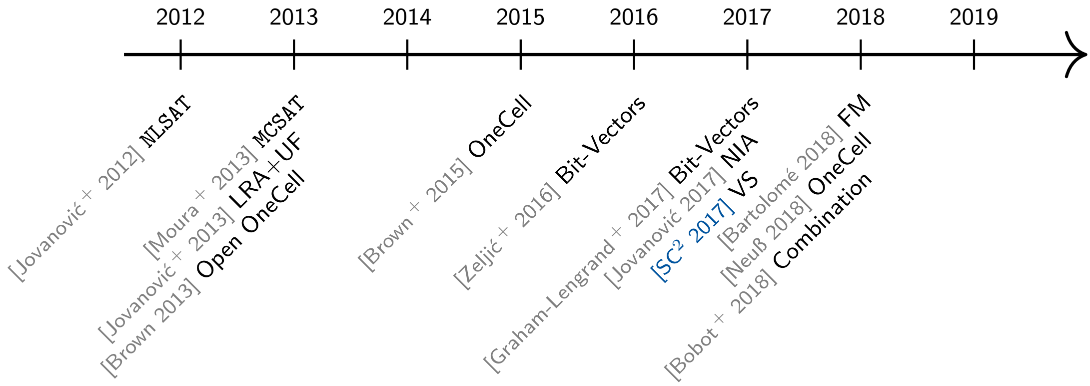

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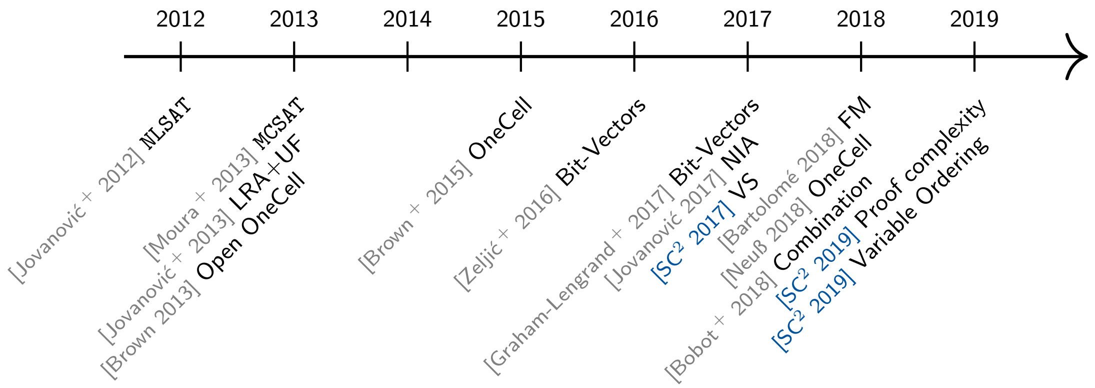

Theory decisions

Regular assignment finder

- Collect univariate constraints
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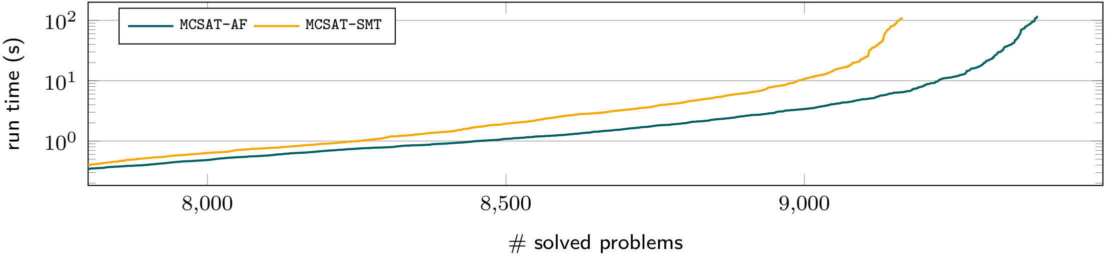

Comparison of CDCL*(T) and MCSAT [SC[2] 2019]

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*: terms and conditions apply